



Heat and mass transfer by Natural convection in a doubly stratified porous medium saturated with Power-law fluid

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Abstract: This paper studies the effects of natural convection heat and mass transfer through a vertical plate embedded in a power-law fluid saturated Darcy's porous medium in addition to double stratification. By using similarity transformations, the governing partial differential equations are transformed into ordinary differential equations and consequently solved using shooting method. Taking into account the physical parameters involved in a problem, a parametric study is carried out and the representative set of numerical results are graphically illustrated.

I. INTRODUCTION

In view of their extensive occurrence in many realistic situations, heat and mass transfer problems in fluid systems have concerned the attention of many researchers for a long time. Even the initial focus was on viscous fluids [1], interest has been increased in non-Newtonian fluids [2-5]. This has been aggravated by many industries (molten plastics, polymeric liquids, foodstuffs). Among all, the power- law fluid has gained its significance which gives an experiential relationship between the stress and velocity gradients [6-7]. In view of significance, Zheng et al. [8] studied the influence of chemical reaction on power-law fluid flow through a porous medium with heat generation. Most recently, Xia et al. [9] presented the nature of poiseuille induced flow of power-law fluids in a circular cylinder.

The impact of stratification is significant in heat and mass transfer analysis. The variations in temperature or the differences in concentration, or the presence of different fluids of different densities causes the formation of layers called stratification. Several investigations have been reported in the literature on the influence of stratification in fluid saturated porous medium. Lare [10] investigated the effects of stratification on Casson fluid flow with variable viscosity and thermal conductivity over an exponential stretching sheet. Srinivasacharya et al. [11] analyzed

the nature of free convection boundary layer flow over a vertical surface in a doubly stratified fluid-saturated porous medium in the presence of constant suction. Hayat et al. [12] reported the influence of thermal radiation and stratification process on unsteady viscous nano fluid flow caused by an inclined stretching sheet. Srinivasacharya and Surender [13] considered the effects of double stratification and thermal radiation on the natural convective flow in a non-Darcy porous medium. Reddy et al. [14] addressed the impact of double stratification on MHD three dimensional Casson nano fluid flow over a stretching sheet. Most recently, Srinivasacharya and Jagadeeshwar [15] studied the impact of double stratification, Cross-Diffusion and Hall currents on the flow over an exponentially stretching sheet.

The earlier literature clearly exhibits that the present study has not been reported elsewhere. In view of applications and significance, the authors are motivated to take this present study on double stratifications on free convection flow saturated power-law fluid from a vertical plate though darcy porous medium.

II. MATHEMATICAL FORMULATION

Here, steady incompressible laminar two dimensional free convection flow along a vertical flat plate in a

power-law fluid through a Darcy porous medium. The plate is maintained at variable surface heat flux $q_w(x)$ and mass flux $q_m(x)$. The temperature and concentration of the ambient medium are $T_\infty(x)$ and $C_\infty(x)$ respectively. It has been assumed that the fluid properties are constant density variation in the Boussinesq approximation. Thermal and solutal stratification effects are considered and the ambient

medium is vertically non-linearly stratified with respect to both temperature and concentration in the form of $T_\infty(x) = T_{\infty,0} + Gx^l$ and $C_\infty(x) = C_{\infty,0} + Hx^m$ respectively, where G and H are constants and varied to alter the intensity of stratification in the medium. Thus we have the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u^n = \frac{gK}{\nu} (\beta_T(T - T_\infty) + \beta_C(C - C_\infty)) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} \quad (4)$$

With

$$v = 0, \quad -k \frac{\partial T}{\partial y} = q_w(x), \quad -D_m \frac{\partial C}{\partial y} = q_m(x) \quad \text{at } y = 0 \text{ and } u \rightarrow 0, T \rightarrow T_\infty(x), C \rightarrow C_\infty(x) \text{ as } y \rightarrow \infty \quad (5)$$

where u and v are the Darcian velocity components in x and y directions, T is the temperature, C is the concentration, k_T is the thermal diffusion ratio, ν is the kinematic viscosity, K is the permeability, g is the acceleration due to gravity, β_T and β_C are the thermal and solutal expansions, α_m is the thermal diffusivity, D_m is the mass diffusivity of the porous medium, and n is the power-law index. When $n = 1$, the Eq. (2) represents a Newtonian fluid. Therefore, deviation of n

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (6)$$

Substituting (6) in (2), (3) and (4) and then using the following similarity transformations

$$\left. \begin{aligned} \psi &= A x^{2/3} f(\eta), \quad \eta = B y x^{-1/3} \quad T = T_\infty(x) + \frac{q_w(x)}{k} \theta(\eta) \\ \frac{q_w(x)}{k} &= E x^{n/3}, \quad C = C_\infty(x) + \frac{q_m(x)}{D} \phi(\eta), \quad \frac{q_m(x)}{D} = F x^{n/3} \end{aligned} \right\} \quad (7)$$

We get the following nonlinear system of differential equations.

$$(f')^n = (\theta + N\phi) \quad (8)$$

$$\theta'' = \frac{1}{3} (n f' \theta - 2 f \theta' + \varepsilon_1 f') \quad (9)$$

$$\phi'' = \frac{Le}{3} (n f' \phi - 2 f \phi' + \varepsilon_2 f') \quad (10)$$

Where primes denote differentiation with respect to η ,

$\varepsilon_1 = \frac{nG}{E}$ and $\varepsilon_2 = \frac{nH}{F}$ are the thermal and solutal

from a unity indicates the degree of deviation from Newtonian behavior. The fluid is shear thinning and thickening when $n < 1$ and $n > 1$ respectively.

III. SOLUTION OF THE PROBLEM

In view of the continuity eq. (1), we introduce the stream function ψ by

stratification parameters respectively, $N = \frac{\beta_C F}{\beta_T E}$ is the

$Le = \frac{\alpha_m}{D_m}$ is the Lewis number. buoyancy ratio, is the

$$A = \left[\frac{Eg K \beta_T \alpha_m^n}{\nu} \right]^{\frac{1}{2n}} \quad \text{and} \quad B = \left[\frac{Eg K \beta_T}{\nu \alpha_m^n} \right]^{\frac{1}{2n}}$$

The boundary conditions (5) in terms of f , θ , and ϕ become

$$f(0) = 0, \quad \theta'(0) = -1, \quad \phi'(0) = -1, \quad f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0$$

The expressions of Nusselt and Sherwood numbers are given by

$$\frac{Nu_x}{Bx^{2/3}} = \frac{1}{\theta'(0)} \quad \text{and} \quad \frac{Sh_x}{Bx^{2/3}} = \frac{1}{\phi'(0)}$$

IV. RESULTS AND DISCUSSION

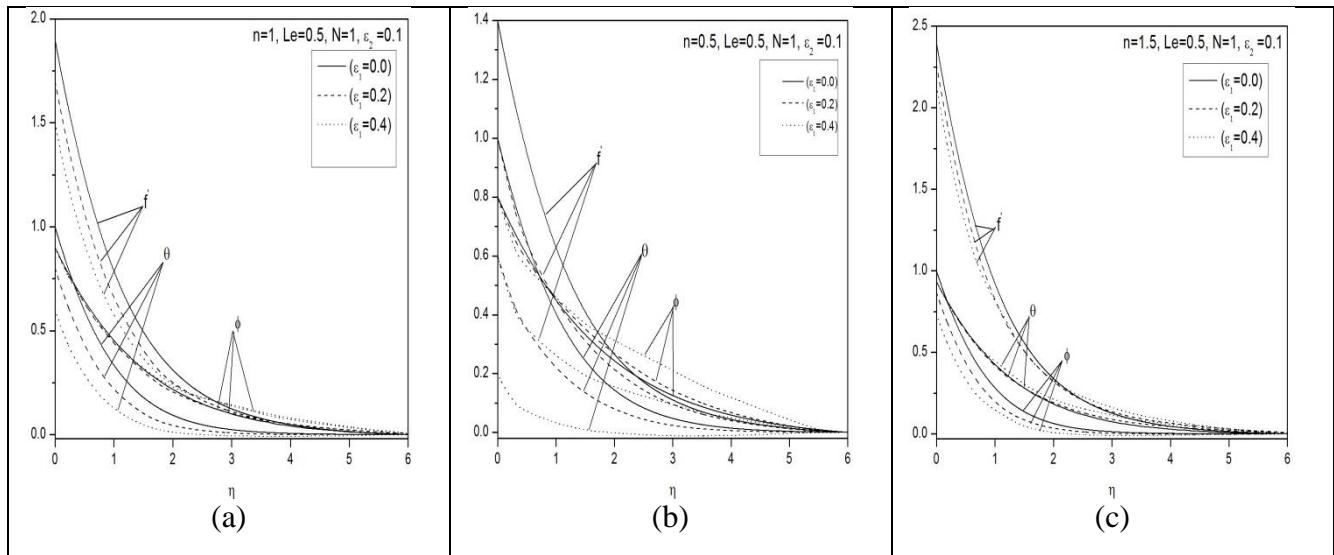


Figure 1: Velocity, Temperature and Concentration profiles for various values of ε_1 for
(a) Pseudo-plastic fluids, (b) Newtonian and (c) dilatant fluids.

The non-dimensional velocity, temperature and concentration are plotted for $N = 1$, $Le = 0.5$, $\varepsilon_2 = 0.1$ in Figs 1(a)-1(c) with varying thermal stratification parameter by considering pseudo-plastic ($n=0.5$), Newtonian ($n=1$) and dilatant fluids. Figs 1(a)-

1(c) demonstrates that the velocity and temperature of the fluid decreased with increasing the value of thermal stratification parameter where as the concentration of the fluid is increased.

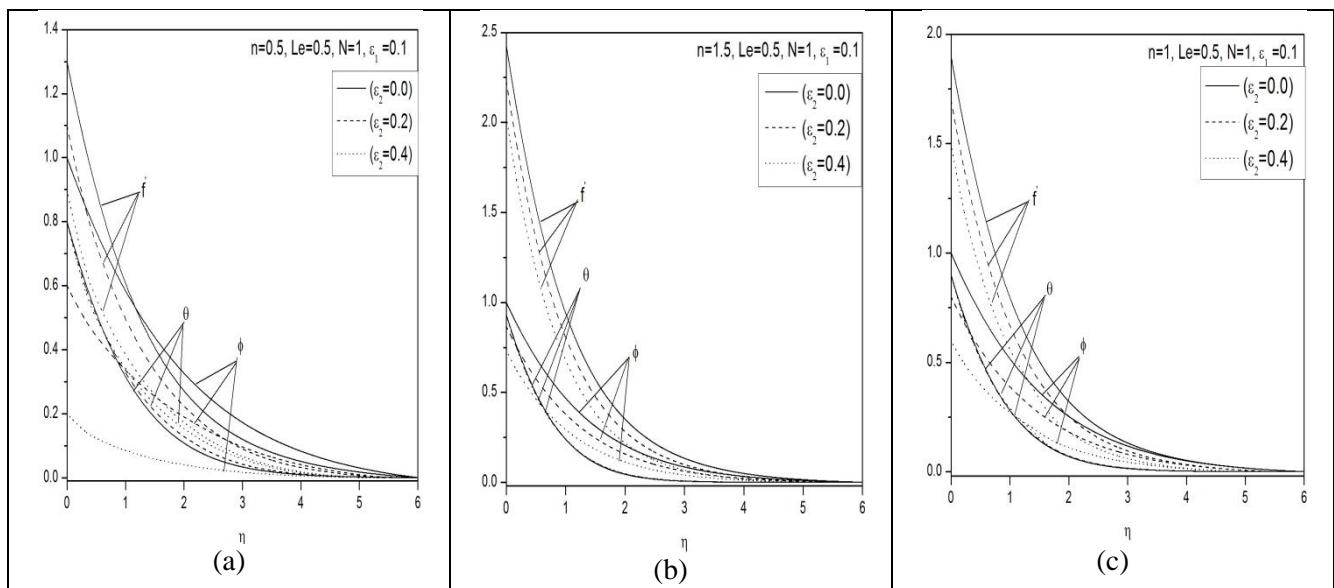


Figure 2: Velocity, Temperature and Concentration profiles for various values of ε_2 for (a) Pseudo-plastic fluids, (b) Newtonian and (c) dilatant fluids.

The non-dimensional velocity, temperature and concentration are plotted for $N = 1$, $Le = 0.5$, $\varepsilon_1 = 0.1$ in Figs2 (a)-2(c) with varying solutal stratification parameter by considering pseudo-plastic ($n=0.5$), Newtonian ($n=1$) and dilatant fluids. It can be observed from Figs2 (a)-2(c) that the velocity and concentration of the fluid decreased whereas the temperature of fluid is increased with increasing the value of solutal stratification parameter.

V. CONCLUSIONS

In this study, free convection power-law fluid flow with heat and mass transfer along a vertical plate embedded in a saturated Darcy porous medium. The influence of thermal and solutal stratification parameters has been studied. It is seen from the present study that the higher values of thermal and solutal stratification parameter result in lower velocity. Temperature of the fluid decreases and concentration increases with an increase in thermal stratification. Whereas there is a reverse trend noticed incase of concentration stratification parameter.

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