



A Study of Thermal Convection in a Horizontal Porous Layer with Effect of Variable Gravity and Internal Heat Source

¹Anjanna Matta

¹Department of Mathematics, Faculty of Science & Technology,
ICFAI Foundation for Higher Education, Hyderabad 501203, Telangana, India
¹anjireddyiith@ifheindia.org

Abstract: In this study, analyze the thermal convection with the effect of heat source and variable gravity in an infinite length of horizontal porous layer. Both the boundaries are considered to be inclined temperature gradients with bottom wall heating and top wall cooling. For a better understanding the combined influence of variable gravity and heat source is investigated by using linear stability analysis. This thermal study is evaluated by applying shooting and Runga-Kutta methods for the velocity, temperature and vertical thermal Rayleigh number (R_z) corresponding to different flow parameters. Increasing in the horizontal Rayleigh number, which caused the strong stabilization irrespective of gravity and mass flow effects. The asymmetric nature velocity and temperature profiles are observed due to enhance the heat source.

Keywords: Thermal convection, variable gravity, heat source, horizontal porous layer.

I. INTRODUCTION

The theoretical study of thermal convection induced by an internal heat source and mass flow is a source of interest for many researchers, in the last few decades, due to its important applications in daily existence. In this analysis the additional effect of gravity in a horizontal porous layer is investigated. It has many real life practical applications are geophysical problems, underground transport of pollutants, gas reservoirs, crystal growth, environmental problems and cooling of nuclear reactors (Chen and Li, 2014; Kwon et al., 2014) etc.

A detailed survey of theoretical analysis and experimental investigations on thermal convection in porous medium can be found in the recent book of Nield and Bejan (2013). Nowadays, studying the effect of gravity on fluid flow in the presence of porous medium has brought attention by many researchers. The influence of gravity modulation in a fluid medium was first analyzed by Gresho and Sani (1970). Different problems of gravity-driven flows in a porous layer are described by Bear (1988). Due to the horizontal temperature gradient a thermogravitational flow occurs, which is known as Hadley circulation. For the first time an overview of the Hadley circulations

has been discussed by Hart (1972 & 1983). Thermal convection in a horizontal porous layer with the inclined temperature gradient has been investigated by Nield (1991 & 1994). Further, Manole and Lage (1995) have been studied both the Hadley-flow and evolution of heat transfer beyond supercritical regimes. The influence of variable gravity field on the Hadley-flow induced by inclined thermal gradient is investigated by Alex and Patil (2002).

Several authors written report on convection as a result of internal heat sources. Few papers of Schwiderski et al. (1971), tveitereid (1977), Yoo and Schultz (2003) and Tritton et al. (1967) are pertaining to the experimental investigations. Roberts (1967) and Thirlby (1970) did a theoretical analysis on the results of Schwiderski et al. (1971) and Tritton et al. (1967). Parthiban and Patil (1995) studied the thermal convection because of nonuniform heating along the boundaries in addition to inclined temperature gradients, after that, they study the extension of anisotropic porous bed in their article Parthiban and Patil (1997). Articles of Alex and Patil (2002) are to be referred for general theory and daily life applications. Matta et al. (2016) and Matta (2019) studied the heat source effect on a horizontal porous layer. Recently Matta (2019) investigated the Hadley-Prats flow in a

horizontal porous layer.

The main aim of the current theoretical examination is to analyze the effects of heat source, variable gravity, mass flow and inclined temperature gradient in a horizontal porous bed. Equations of the flow are converted into an eigenvalue problem. Later, it's solved, to determine the various effects of thermal convection and instability in a horizontal porous layer.

II. MATHEMATICAL FORMULATION FORMAT

Let us consider the horizontal fluid-saturated porous layer with height d , where the z' axis is vertical with heat source Q' and $\Delta\theta$ is vertical thermal differences along the boundaries. Consider M is the net mass flow along the x' axis is shown in Fig. 1. The imposed thermal gradient (β_{θ_x}). Since the flow passes through porous layer and fluid varies in temperature from place to place. It follows the Darcy law and linear Boussinesq approximation. Hence, the governing equations of the flow field in non-dimensional form are

$$\nabla \cdot \mathbf{q} = 0 \quad (1)$$

$$(x, y, z) = \frac{1}{d}(x', y', z'), \quad t = \frac{\alpha_m t'}{ad^2}, \quad q = \frac{d q'}{\alpha_m}, \quad P = \frac{K(P' - \rho_0 \int g dz')}{\mu \alpha_m},$$

$$\theta = \frac{R_z(\theta' - \theta_0)}{\Delta T}, \quad Q = \frac{d^2 Q'}{k_m \Delta \theta},$$

$$\gamma = \frac{\alpha_m \gamma'}{\alpha d^2}, \quad \alpha_m = \frac{k_m}{(\rho c_p)_f}, \quad a = \frac{(\rho c)_m}{(\rho c_p)_f}$$

Where

$$R_x = \frac{\rho_0 g \gamma \theta K d^2 \beta_{\theta_x}}{\mu \alpha_m}, \quad R_z = \frac{\rho_0 g \gamma \theta K d \Delta \theta}{\mu \alpha_m} \quad (6)$$

Here, R_x denotes thermal Rayleigh number along the x' axis and R_z denotes vertical thermal Rayleigh number, it is defined as follows

$$R_x = \frac{\rho_0 g \gamma \theta K d^2 \beta_{\theta_x}}{\mu \alpha_m}, \quad R_z = \frac{\rho_0 g \gamma \theta K d \Delta \theta}{\mu \alpha_m} \quad (6)$$

2.1. Steady-State Solution

Basic state solution of the equations (1) - (3), with the conditions (4) is as follows

$$\theta_s = \tilde{\theta}(z) - R_x x, \quad (7)$$

$$(u_s, v_s, w_s) = (u(z), 0, 0), \quad P_s = P(x, y, z) \quad (8)$$

with

$$u_s = -\frac{\partial P}{\partial x}, \quad v_s = -\frac{\partial P}{\partial y}, \quad (9)$$

$$\mathbf{q} + \nabla P - \theta(1 + \gamma z) \mathbf{k} = 0 \quad (2)$$

$$\frac{\partial \theta}{\partial t} + q \cdot \nabla \theta = \nabla^2 \theta + QR_z \quad (3)$$

Boundary conditions at walls:

$$w = 0, \quad \theta = -\frac{1}{2}(\pm R_z) - R_x x \quad \text{at} \quad z = \pm \frac{1}{2} \quad (4)$$

$$w' = 0, \quad \theta' = \theta_0 - \frac{1}{2} \Delta \theta - \beta_{\theta_x} x' \quad (4)$$

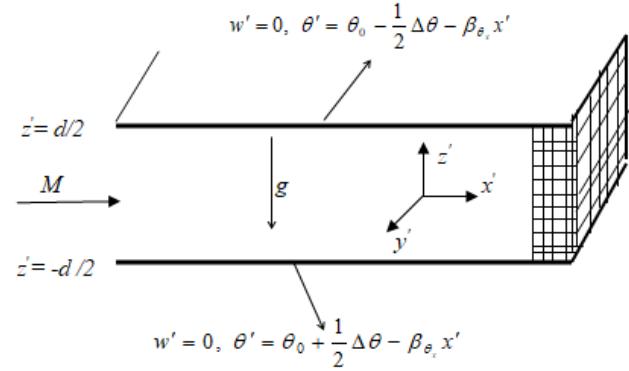


Figure 1: Physical system
 Dimensionless variables are given below

$$0 = -\frac{\partial P}{\partial z} + (1 + \gamma z)(\tilde{\theta}(z) - R_x x), \quad (10)$$

$$D^2 \tilde{\theta}(z) = -QR_z - u_s R_x \quad (11)$$

Where, $D = \frac{d}{dz}$, here, the net flow is $\int_{-0.5}^{0.5} u(z) dz = M$, along the x -axis and $\int_{-0.5}^{0.5} v(z) dz = 0$ along the y -axis. The steady state solution appeared in the form of flow seepage velocity and temperature is obtained.

$$u_s = R_x \left(z + \gamma \frac{z^2}{2} - \frac{\gamma}{24} \right) + M, \quad v_s = 0, \quad w_s = 0, \quad (12)$$

$$\begin{aligned} \bar{\theta} = & -R_z z + \frac{R_x^2}{24} \left(z - 4z^3 + \frac{\gamma}{16} (4z^2 - 1)^2 \right) \\ & + \frac{(QR_z + MR_x)}{8} (1 - 4z^2), \end{aligned} \quad (13)$$

2. 2. Linear Stability Analysis

Let us consider the following perturbed solution $q = q_s + \bar{q}$, $\theta = \theta_s + \bar{\theta}$ and $P = P_s + \bar{P}$. Following

$$\nabla \theta_s = - \left(R_x, R_y, R_z - \frac{R_x^2}{24} [1 - 12z^2 - \gamma(4z^2 - 1)] - (QR_z + MR_x)z \right)$$

Where,

The boundary conditions at walls are

$$\bar{w} = 0, \bar{\theta} = 0 \text{ at } z = \pm 0.5. \quad (17)$$

These conditions in Eq. (17) says that, perturbed velocity and perturbed temperature vanished at the walls. Thereafter, solution of Eqs. (14) - (16) is found in the form of normal modes using the following

$$[\bar{q}, \bar{P}, \bar{\theta}] = [q(z), P(z), \theta(z)] \exp\{i(kx + ly) - \sigma t\} \quad (18)$$

Later, we get the following equations after elimination of P from the equation (15).

linear system is obtained from the equations (1) - (3) after substituting the above perturbed solution and after neglecting the nonlinear terms.

$$\nabla \cdot \bar{\mathbf{q}} = 0 \quad (14)$$

$$\bar{\mathbf{q}} = -\nabla \bar{P} + (1 + \gamma z) \bar{\theta} \mathbf{k} \quad (15)$$

$$\frac{\partial \bar{\theta}}{\partial t} + q_s \cdot \nabla \bar{\theta} + \bar{q} \cdot \nabla \theta_s = \nabla^2 \bar{\theta} \quad (16)$$

$$(D^2 - \alpha^2)w + \alpha^2(1 + \gamma z)\theta = 0 \quad (19)$$

$$\begin{aligned} & (D^2 - \alpha^2 + i(\sigma - ku_s - lv_s))\theta \\ & + \frac{i}{\alpha^2}(kR_x)Dw - (D\bar{\theta})w = 0 \end{aligned} \quad (20)$$

The above Eqn. (19) - (20) together with conditions $w = \theta = 0$ at upper plate $z = 0.5$ and lower plate $z = -0.5$ where γ , M , Q , R_x , k and l are parameters, it is an eigenvalue problem in terms of vertical thermal Rayleigh number R_z . Here, the overall wave number $\alpha = \sqrt{k^2 + l^2}$.

Table 1: Vertical thermal Rayleigh number at $Q = 0$ and $M = 0$

	R_x	0	10	20	30	40	50	60
$\gamma = 0$	R_z	39.4784	42.0076	49.5486	61.9566	78.9663	100.1163	124.4724
	α	3.1399	3.1399	3.1499	3.1599	3.2199	3.3399	3.6699
$\gamma = 1$	R_z	39.2368	41.8353	49.5842	62.3395	79.8415	101.6537	126.9460
	α	3.1599	3.1499	3.1499	3.1599	3.1999	3.30999	3.56499
$\gamma = 2$	R_z	38.5398	41.3396	49.6931	63.4610	82.40460	106.1582	134.1289
	α	3.1999	3.1899	3.1799	3.1599	3.1699	3.2099	3.3499

III. RESULTS AND DISCUSSION

In this work analyzed the thermal convection in a horizontal fluid-saturated porous bed with the effect of an internal heated and variable gravity. The classical normal mode technique is used to study the thermal instability of Hadley-Prats flow. Here, the minimum of all R_z values is defined as the vertical thermal Rayleigh number R_z as the wave number (α) varies. Stationary convection is achieved by putting $\sigma = 0$ as in Nield (1991). The term longitudinal disturbance is characterized by $k = 0$. From the Table 1, it is noticed

that the results are in very good agreement and comparison with the past results of the article Nield (1994), when $Q = 0$, $M = 0$ and $\gamma = 0$. Also, the critical values of R_z increased if the value of γ increases from 0 to 2, shown in Table 1. Thus, strong stabilization is the result of the flow field is observed in the medium.

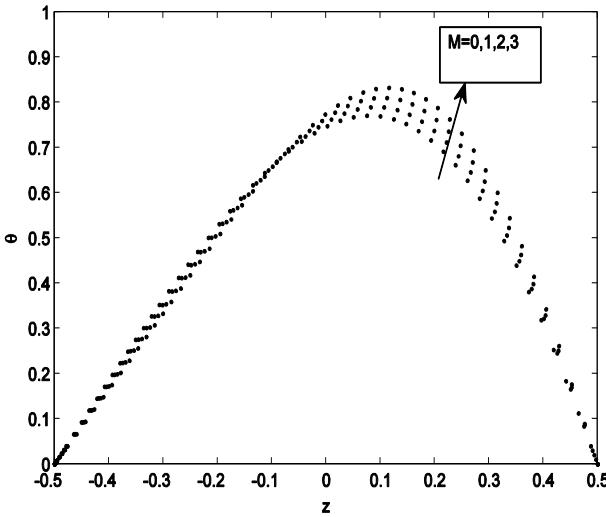


Figure 2: Temperature contours θ with z by varying mass flow (M)

Figs. 2 - 3 shows the temperature and velocity contours by varied the mass flow (M) at $R_x = 10$ and $Q = 1$. It is noted that as an increase in the mass flow (M) from 0 to 3, the temperature and velocity profiles enhanced middle of the porous layer is observed. Hence, the temperature and velocity of the fluid are increased. The velocity and temperature profiles also observed symmetric structure.

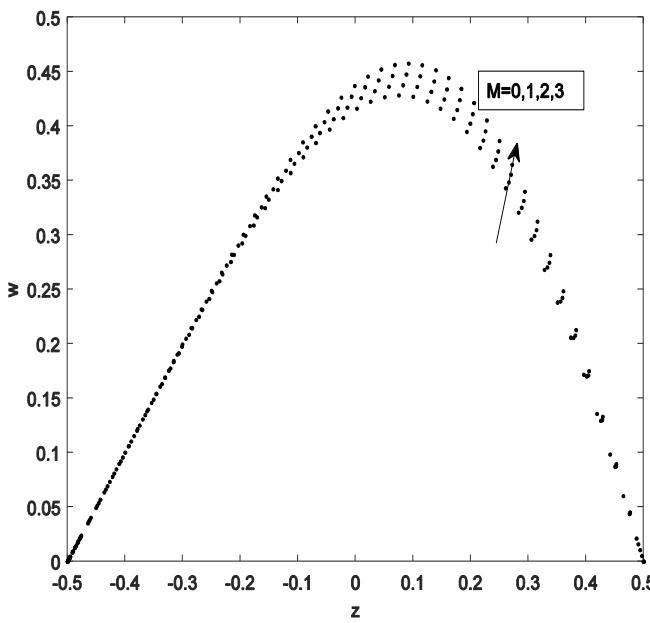


Figure 3: Velocity profiles (w) with z by varying mass flow (M)

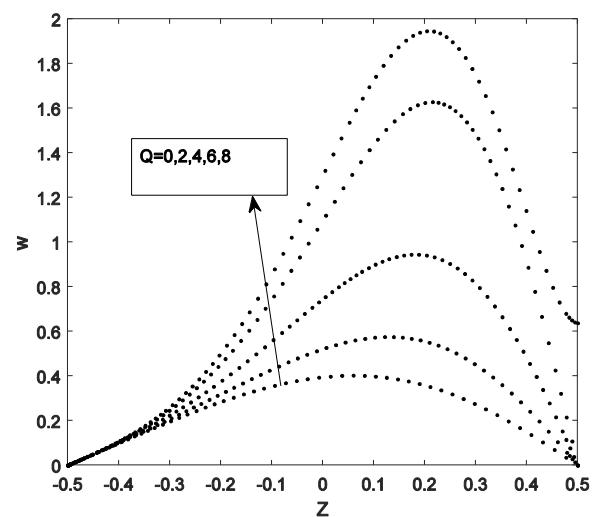


Figure 4: Velocity profiles w with z by varying the heat source (Q)

Figs. 4 - 5 shows the velocity and temperature profiles by varying the heat source (Q) at $R_x = 10$ and $M = 0$. It is noted that as an increase in the heat source (Q) from 0 to 8, the temperature value also increased middle of the porous bed is observed. It is indicated that, the flow moves from bottom to top layer as increased the heat source. Hence, the velocity and temperature of fluid is increased, due to enhance the global temperature of the system. The velocity and temperature profiles symmetric structure also distributed. It is caused to due to enhance the heat source.

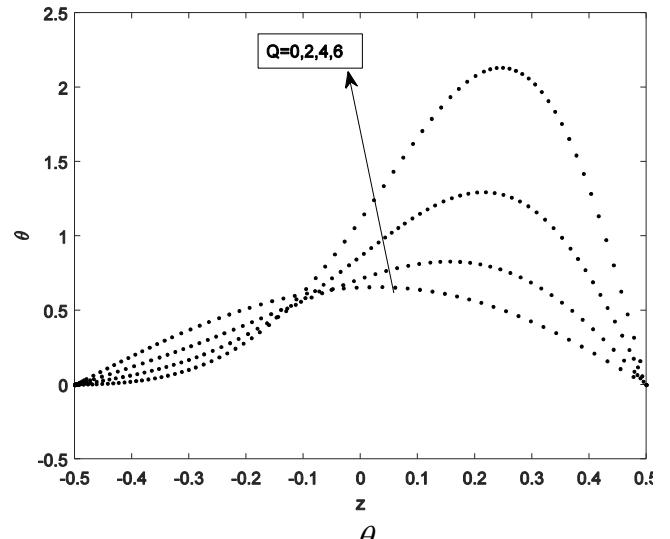


Figure 5: Temperature contours θ with z by varying heat source (Q)

IV. CONCLUSION

The present study used the stability analysis to analyze the thermal convention of a Hadley-Prats flow in a horizontal porous bed. It has also studied the influence of internal heat generation and variable gravity effects

on velocity and temperature fields. The thermal Rayleigh number (R_z) computed for different combinations of parameters governing the flow. It has been concluded as follows.

(i) Enhance in the heat source results causes to improve the temperature in the horizontal porous layer irrespective of gravity and mass flow.

(ii) Strong stabilization of the flow field is observed at a higher horizontal Rayleigh number and in the existence of gravity.

(iii) The effect of heat source clearly observed by the asymmetric nature velocity and temperature profiles subject to mass flow and variable gravity.

Acknowledgements

The author is grateful to the anonymous referees for a remark which is improved the work considerably.

Conflict of Interest

Both the authors have equal contribution in this work and it is declared that there is no conflict of interest for this publication.

References

- [1] Alex, S. M., & Patil, P.R. (2002). Effect of a variable gravity field on convection in an anisotropic porous medium with internal heat source and inclined temperature gradient, *Transactions of the ASME*.
- [2] Alex , S. M. & Patil, P. R. (2002). Effect of a variable gravity field on thermal instability in a porous medium with inclined temperature gradient and vertical through flow, *J. Porous Media*, 5.
- [3] Bear, J. (1988). *Dynamics of Fluids in Porous Media*, Dover.
- [4] Chen, Xi & Li, A. (2014). An experimental study on particle deposition above near-wall heat source, *Building and Environment*, 81.
- [5] Kwon, O., Bae, K. & Park, C. (2014). Cooling characteristics of ground source heat pump with heat exchange methods, *Renewable Energy*, 71.
- [6] Gresho, P. M. & Sani, R. L. (1970). The effect of gravity modulation on the stability of a heated fluid layer, *J. Fluid Mech.*, 40.
- [7] Hart, J. E. (1972). Stability of thin non-rotating Hadley circulations, *J. Atmos. Sci.*, 29.
- [8] Hart, J. E. (1983). A note on the stability of low-Prandtl-number Hadley circulations, *J. Fluid Mech.*, 132.
- [9] Manole, D. M. & Lage, J. L. (1995). Numerical simulation of supercritical Hadley circulation within a porous layer induced by inclined temperature gradients, *Int. J. Heat Mass Transfer*, 38.
- [10] Matta, A., Narayana, P. A. L. & Hill, A. A. (2016). Non-linear thermal instability in a horizontal porous layer with an internal heat source and mass flow, *Acta Mechanica*.
- [11] Matta, A. (2019). On the stability of hadley-flow in a horizontal porous layer with non-uniform thermal gradient and internal heat source, *Microgravity Science and Technology*, 31 (2), 169–175.
- [12] Matta, A. (2019). Mono-diffusive Hadley–Prats flow in a horizontal porous layer subject to variable gravity and internal heat generation, *Heat Transfer - Asian Research*, 48(4), 1399-1412.
- [13] Nield, D.A., (1991). Convection in a porous medium with inclined temperature gradient, *Int. J. Heat Mass Transfer*, 34.
- [14] Nield, D.A., (1994). Convection induced by an inclined temperature gradient in a shallow horizontal layer, *Int. J. Heat and Fluid Flow*, 15.
- [15] Nield, D. A. (1994). Convection in a porous medium with inclined temperature gradient: additional results, *Int J. Heat Mass Transfer*, 37.
- [16] Nield, D. A. & Bejan, A. (2013). *Convection in Porous Media*, forth ed., Springer .
- [17] Parthiban, C. & Patil, P.R. (1995). Effect of non-uniform boundary temperatures on thermal instability in a porous medium with internal heat source, *Int. Commun. Heat Mass Transfer*, 22.
- [18] Parthiban, C. & Patil, P. R. (1997). Thermal instability in an anisotropic porous medium with internal heat source and inclined temperature gradient, *Int. Comm. Heat Mass Transfer*, 24.
- [19] Roberts, P. H. (1967). Convection in horizontal layers with internal heat generation: Theory *J. Fluid Mech.*, 30.
- [20] Schwiderskei, W. & Schwab, J. A. (1971). Convection experiments with electrolytically heated fluid layers, *J. Fluid Mech.*, 48.
- [21] Thirlby, R., (1970). Convection in an internally heated layer. *J. Fluid Mech.*, 44.
- [22] Tritton, J. & Zarraua, M. N. (1967). Convection in horizontal layers with internal heat generation, *J. Fluid Mech.*, 30.
- [23] Tveitereid, M. (1977). Thermal convection in a horizontal porous layer with internal heat sources, *Int. J. Heat Mass Transfer*, 20.
- [24] Yoo, J. S. & Schultz, W. W. (2003). Thermal convection in a horizontal porous layer with spatially periodic boundary temperatures: small Ra flow, *Int. J. of Heat and Mass Transfer*, 46.