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A Novel Numerical Technique for Free Convective Heat and Mass Transfer under Mixed Thermal Boundary Condition

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Abstract: *This work deals with a novel numerical technique for similarity analysis of the effect of magnetic field on free convection in micropolar fluid about a vertical strip in the existence of mixed thermal boundary conditions and non-uniform wall concentration boundary conditions. The governing boundary layer equations of the physics of the present problem are cast into coupled nonlinear ordinary differential equations using the Lie Group point transformations. These resulted equations engaging in* $\boldsymbol{m}$ *which speculates the values* $\boldsymbol{zero}, \boldsymbol{one}$ *and* $\boldsymbol{infinity}$ *leads to the prescriptive temperature case, prescriptive heat flux case and radiation boundary conditions case at wall, has been solved using the novel numerical technique, paired quasi-linearisation method (PQLM). The results obtained are corroborated and are establish to be matched with earlier works.*

Keywords: Mixed Thermal Boundary Conditions, Micropolar Fluid, MHD, Paired QLM

I. INTRODUCTION

(Sparrow and Gregg, 1956) have pioneered the problem of the convective mechanism of heat transfer through heated surface of a vertical plate. Later a great deal of attention has been drawn by many researchers on the problem of free convection next to heated vertical surface. These prior analyses of flow of the boundary–layer of free-convection along a surface which is heated have simulated under either prescriptive temperature or heat flux on the wall conditions. The problem of free convection has not acknowledged abundant consideration when the surface exposes to a mixed thermal boundary condition. As a result of the small difference in the temperature between the solid surface and the medium within the vicinity of the surface, the radiation type mechanism of heat transfer takes place from surface to the medium. This radiation heat transfer at the boundary of the plate and medium is known as the radiation boundary condition. In the present work, for the case $m \to \infty$, the mixed thermal boundary condition reduces to the RBC. (Ramanaiah and Malarvizhi, 1992) have been studied the convective flow on the wedge surface as well as on the surface of a cone for the case of radiation type thermal boundary condition. (Ece, 2005) investigated the transverse magnetic effect on the flow of convective laminar boundary – layer under the radiation type thermal boundary condition. (Cheng, 2009) studied the steady natural convection in porous media along a downward pointing vertical cone submerged in the non-Newtonian power–law fluid subjected to the mixed thermal boundary condition. The intent of the present work is to acquire profiles of velocity, microrotation, temperature and concentration though the similarity solutions under the mixed thermal boundary condition.

II. MATHEMATICAL FORMULATION

Grant a free convective, incompressible and steady micropolar fluid along a vertical plate. Fix the vertical plate along the $\bar{x}-$axis and consider the $\bar{y}-$axis normal to the plate. T_∞ is the ambient temperature and C_∞ is the ambient concentration. The equations of the physics of the present geometry after employing the assumption of laminar boundary-layer flow along with the approximation of Boussinesq, are:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = \frac{\mu+\kappa}{\rho}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\kappa}{\rho}\frac{\partial \bar{\omega}}{\partial \bar{y}} - \frac{\sigma B_0{}^2}{\rho}\bar{u} + g^*\left(\beta_T\left(\bar{T}-T_\infty\right)+\beta_C\left(\bar{C}-C_\infty\right)\right) \quad (2)$$

$$\bar{u}\frac{\partial \bar{\omega}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{\omega}}{\partial \bar{y}} = \frac{\gamma}{j\rho}\frac{\partial^2 \bar{\omega}}{\partial \bar{y}^2} - \frac{\kappa}{j\rho}\left(2\bar{\omega} + \frac{\partial \bar{u}}{\partial \bar{y}}\right) \quad (3)$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \quad (4)$$

$$\bar{u}\frac{\partial \bar{C}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \quad (5)$$

where $\bar{u}$ and $\bar{v}$ are the components of velocity along $\bar{x}$ and $\bar{y}$ directions respectively, $\bar{\omega}$ is the microrotation component, g^* is the gravitational acceleration, $\bar{T}$ is the dimensional temperature, $\bar{C}$ is the dimensional concentration, coefficients of expansions of thermal and solutal are given by β_T and β_C, μ is the dynamic viscosity coefficient of the fluid, κ stands for vortex viscosity, γ is the spin-gradient viscosity, σ denotes the permeability of the magnetic of the fluid, ν represents the viscosity of kinematic, α is the thermal diffusivity, D is the molecular diffusivity. We consider the assumption that $\gamma = (\mu + \kappa/2)j$ which is mentioned in the work of many recent authors. This assumption is invoked to allow the field of equations predicts the correct behaviour in the limiting case when the microstructure effects become negligible and the total spin $\bar{\omega}$ reduces to the angular velocity. The boundary conditions are:

$$\left.\begin{array}{ll} \bar{u}=0,\ \bar{v}=0,\ \bar{\omega}=0,\ \bar{C}=\bar{C}_w(\bar{x}) & at\ \bar{y}=0 \\ \bar{u}\to 0,\ \bar{\omega}\to 0,\ \bar{T}\to T_\infty,\ \bar{C}\to C_\infty(\bar{x}) & as\ \bar{y}\to\infty \end{array}\right\} \quad (6)$$

The generalized form of the mixed thermal boundary condition on the surface of the vertical plate $\bar{y}=0$ is assumed to be:

$$a_0(\bar{x})\left(\bar{T}-T_\infty\right)_{\bar{y}=0} - a_1(\bar{x})\frac{\partial \bar{T}}{\partial \bar{y}} = a_2(\bar{x}) \quad (7)$$

where, the subscripts w and ∞ indicates the conditions at wall and at the outer edge of the boundary layer, respectively.

III. METHOD OF SOLUTION

Introducing the following non-dimensional variables

$$\left.\begin{array}{l} x=\frac{\bar{x}}{L},\ y=\frac{\bar{y}}{L}Gr^{1/4},\ u=\frac{\bar{u}L}{\nu Gr^{1/2}},\ v=\frac{\bar{u}L}{\nu Gr^{1/4}},\ \omega=\frac{\bar{\omega}L^2}{\nu Gr^{3/4}},\ Gr=\frac{g^*\beta_T\Delta TL^3}{\nu^2}, \\ \theta=\frac{\bar{T}-T_\infty}{\bar{T}_w(\bar{x})-T_\infty},\ x\Delta T=\bar{T}_w(\bar{x})-T_\infty,\ \phi=\frac{\bar{C}-C_\infty}{\bar{C}_w(\bar{x})-C_\infty},\ x\Delta C=\bar{C}_w(\bar{x})-C_\infty \end{array}\right\} \quad (8)$$

and the stream function ψ through $u=\frac{\partial \psi}{\partial y}$ and $v=-\frac{\partial \psi}{\partial x}$ in to the Eqs. (1) - (5) and (6) - (7) we get

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = (1+K)\frac{\partial^3 \psi}{\partial y^3} + K\frac{\partial \omega}{\partial y} + x(\theta + B\phi) - M\frac{\partial \psi}{\partial y} \quad (9)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} = (1+K/2)\frac{\partial^2 \omega}{\partial y^2} - K\left(2\omega + \frac{\partial^2 \psi}{\partial y^2}\right) \quad (10)$$

$$x\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-x\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}+\theta\frac{\partial\psi}{\partial y}=\frac{1}{\Pr}x\frac{\partial^2\theta}{\partial y^2} \quad (11)$$

$$x\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-x\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}+\phi\frac{\partial\psi}{\partial y}=\frac{1}{Sc}x\frac{\partial^2\phi}{\partial y^2} \quad (12)$$

where $K=\kappa/\mu$ is the coupling parameter, $B=(\beta_C\Delta C)/(\beta_T\Delta T)$ denotes the buoyancy term, $M=(\sigma B_0^2L^3)/(\rho\nu^2 Gr)$ is the magnetic parameter $\Pr=\nu/\alpha$ is the Prandtl number and $Sc=\nu/D$ is the Schmidt number. The transformed boundary conditions are

$$\frac{\partial\psi}{\partial y}=0,\ \frac{\partial\psi}{\partial x}=0,\ \omega=0,\ b_0(x)\Delta T\theta(x,0)-b_1(x)(\Delta T)^{5/4}\theta'(x,0)=1,\ \phi(x,0)=0 \text{ at } y=0 \quad (13)$$

$$\frac{\partial\psi}{\partial y}\to 0,\ \omega\to 0,\ \theta\to 0,\ \phi\to 0 \text{ as } y\to 0 \quad (14)$$

where $b_0(x)=a_0(\bar{x})/a_2(\bar{x})$ and $b_1(x)=a_1(\bar{x})/a_2(\bar{x})$. To compose a solution of the similarity type, these $b_0(x)$ and $b_1(x)$ must be constants and they are denoted to as b_0 and b_1. For disposed values of the constants b_0, T_∞ and b_1 the temperature T of the reference may be chosen to satisfy the equation without any loss of generality, $b_1(\Delta T)^{5/4}+b_0\Delta T-1=0$. By defining $m=b_1(\Delta T)^{5/4}$, the thermal boundary condition at $y=0$ $y=0$ can be written as

$$(1-m)\theta(x,0)-m\theta'(x,0)=1 \quad (15)$$

IV. SIMILARITY SOLUTIONS VIA LIE GROUP ANALYSIS

We now introduce the one-parameter scaling group of transformations which is a simplified form of Lie group transformation

$$\tau:\ \hat{x}=xe^{\varepsilon\alpha_1},\ \hat{y}=ye^{\varepsilon\alpha_2},\ \hat{\psi}=\psi e^{\varepsilon\alpha_3},\ \hat{\omega}=\omega e^{\varepsilon\alpha_4},\ \hat{\theta}=\theta e^{\varepsilon\alpha_5},\ \hat{\phi}=\phi e^{\varepsilon\alpha_6} \quad (16)$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are transformation parameters and ε is a small parameter. Eqs. (9) – (12) and boundary conditions (13) – (15) are invariant under the point transformations (16), and reduce to

$$\left.\begin{aligned}e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\hat{\psi}}{\partial\hat{y}}\frac{\partial^2\hat{\psi}}{\partial\hat{x}\partial\hat{y}}-\frac{\partial\hat{\psi}}{\partial\hat{x}}\frac{\partial^2\hat{\psi}}{\partial\hat{y}^2}\right)=(1+K)e^{\varepsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\hat{\psi}}{\partial\hat{y}^3}+Ke^{\varepsilon(\alpha_2-\alpha_4)}\frac{\partial\hat{\omega}}{\partial\hat{y}}+\\ \hat{x}\left(e^{\varepsilon(\alpha_1-\alpha_5)}\hat{\theta}+Be^{\varepsilon(\alpha_1-\alpha_6)}\hat{\phi}\right)-Me^{\varepsilon(\alpha_2-\alpha_3)}\frac{\partial\hat{\psi}}{\partial\hat{y}}\end{aligned}\right\} \quad (17)$$

$$\left.\begin{aligned}e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_4)}\left(\frac{\partial\hat{\psi}}{\partial\hat{y}}\frac{\partial\hat{\omega}}{\partial\hat{x}}-\frac{\partial\hat{\psi}}{\partial\hat{x}}\frac{\partial\hat{\omega}}{\partial\hat{y}}\right)=\left(1+\frac{K}{2}\right)e^{\varepsilon(2\alpha_2-\alpha_4)}\frac{\partial^2\hat{\omega}}{\partial\hat{y}^2}-\\ K\left(2e^{-\varepsilon\alpha_4}\hat{\omega}+e^{\varepsilon(2\alpha_2-\alpha_3)}\frac{\partial^2\hat{\psi}}{\partial\hat{y}^2}\right)\end{aligned}\right\} \quad (18)$$

$$e^{\varepsilon(\alpha_2-\alpha_3-\alpha_5)}\left(\hat{x}\frac{\partial\hat{\psi}}{\partial\hat{y}}\frac{\partial\hat{\theta}}{\partial\hat{x}}-\hat{x}\frac{\partial\hat{\psi}}{\partial\hat{x}}\frac{\partial\hat{\theta}}{\partial\hat{y}}+\hat{\theta}\frac{\partial\hat{\psi}}{\partial\hat{y}}\right)=e^{\varepsilon(-\alpha_1+2\alpha_2-\alpha_5)}\frac{1}{\Pr}\hat{x}\frac{\partial^2\hat{\theta}}{\partial\hat{y}^2} \quad (19)$$

$$e^{\varepsilon(\alpha_2-\alpha_3-\alpha_6)}\left(\hat{x}\frac{\partial\hat{\psi}}{\partial\hat{y}}\frac{\partial\hat{\phi}}{\partial\hat{x}}-\hat{x}\frac{\partial\hat{\psi}}{\partial\hat{x}}\frac{\partial\hat{\phi}}{\partial\hat{y}}+\hat{\phi}\frac{\partial\hat{\psi}}{\partial\hat{y}}\right)=e^{\varepsilon(-\alpha_1+2\alpha_2-\alpha_6)}\frac{1}{Sc}\hat{x}\frac{\partial^2\hat{\phi}}{\partial\hat{y}^2} \quad (20)$$

and the associated boundary conditions become

$$\left.\begin{array}{ll}\frac{\partial\hat{\psi}}{\partial\hat{y}}=0,\ \frac{\partial\hat{\psi}}{\partial\hat{x}}=0,\ \hat{\omega}=0,\ e^{-\varepsilon\alpha_6}\hat{\phi}=0 & at\quad \hat{y}e^{-\varepsilon\alpha_2}=0\\ \frac{\partial\hat{\psi}}{\partial\hat{y}}\to 0,\ \hat{\omega}\to 0,\ \hat{\theta}\to 0,\ \hat{\phi}\to 0 & as\quad \hat{y}e^{-\varepsilon\alpha_2}\to\infty\end{array}\right\}\tag{21}$$

and the mixed thermal boundary condition at the surface reduces to

$$(1-m)e^{-\varepsilon\alpha_5}\hat{\theta}-me^{\varepsilon(\alpha_2-\alpha_5)}\hat{\theta}'=1\tag{22}$$

Since the group transformations (16) keep the system invariant, hence the relations among the parameters from the Equations (17) – (20) is as below

$$\left.\begin{array}{l}\alpha_1+2\alpha_2-2\alpha_3=3\alpha_2-\alpha_3=\alpha_2-\alpha_4=\alpha_1-\alpha_5=\alpha_1-\alpha_6=\alpha_2-\alpha_3\\ \alpha_1+\alpha_2-\alpha_3-\alpha_4=2\alpha_2-\alpha_4=-\alpha_4=2\alpha_2-\alpha_3\\ \alpha_2-\alpha_3-\alpha_5=-\alpha_1+2\alpha_2-\alpha_5,\qquad \alpha_2-\alpha_3-\alpha_6=-\alpha_1+2\alpha_2-\alpha_6\end{array}\right\}\tag{23}$$

These relations give $\alpha_1=\alpha_3=\alpha_4$ & $\alpha_2=0=\alpha_5=\alpha_6$, so the infinitesimal transformations reduces to point transformations in one parameter as follows

$$\tau:\ \hat{x}=xe^{\varepsilon\alpha_1},\ \hat{y}=y,\ \hat{\psi}=\psi e^{\varepsilon\alpha_1},\ \hat{\omega}=\omega e^{\varepsilon\alpha_1},\ \hat{\theta}=\theta,\ \hat{\phi}=\phi\tag{24}$$

After expanding the Lie group of point transformations in one parameter using the Taylor's series in powers of ε by considering the terms up to $O(\varepsilon)$, we get

$$\hat{x}-x=x\varepsilon\alpha_1,\ \hat{y}-y=0,\ \hat{\psi}-\psi=\psi\varepsilon\alpha_1,\ \hat{\omega}-\omega=\omega\varepsilon\alpha_1,\ \hat{\theta}-\theta=0,\ \hat{\phi}-\phi=0\tag{25}$$

The corresponding characteristic equations are given by

$$\frac{dx}{x\alpha_1}=\frac{dy}{0}=\frac{d\psi}{\psi\alpha_1}=\frac{d\omega}{\omega\alpha_1}=\frac{d\theta}{0}=\frac{d\phi}{0}\tag{26}$$

The self similar solutions of characteristic equations (26) give the similarity transformations as $\hat{y}=\eta,\ \hat{\psi}=\hat{x}f(\eta),\ \hat{\omega}=\hat{x}g(\eta),\ \hat{\theta}=\theta(\eta),\ \hat{\phi}=\phi(\eta)$, these reduce the Eqs. (17) - (20) into

$$(1+K)f'''-(f')^2+ff''+Kg'+\theta+B\phi-Mf'=0\tag{27}$$

$$\left(1+\frac{K}{2}\right)g''+fg'-gf'-K(2g+f'')=0\tag{28}$$

$$\frac{1}{\Pr}\theta''+f\theta'-f'\theta=0\tag{29}$$

$$\frac{1}{Sc}\phi''+f\phi'-f'\phi=0\tag{30}$$

The boundary conditions (21) and (22) in terms of $f,\ g,\ \theta,\ \phi$ now get transformed into

$$f'=0,\ f=0,\ g=0,\ \phi=1\quad at\quad \eta=0,\quad f'\to 0,\ g\to 0,\ \theta\to 0,\ \phi\to 0\ as\ \eta\to\infty\tag{31}$$

and the mixed thermal condition at $\eta=0$ is given by

$$(1-m)\theta(0)-m\theta'(0)=1 \tag{32}$$

The following cases of mixed thermal boundary condition are of special interest: **Case (a):** Variable Wall Temperature (VWT): If $T=T_w(x)$ at $y=0$, where $T_w(x)=T_\infty+\Delta T x^m$ gives $m=0$ thus mixed thermal boundary condition reduces to the form $\theta(0)=1$. **Case (b)**: Variable Wall Heat Flux (VWHF): For this case, the variable wall heat flux condition $-\frac{\partial T}{\partial y}=\Delta T x^m$, gives $m=1$, hence, the mixed thermal boundary condition reduces to $\theta'(0)=-1$. **Case(c)**: Radiation Boundary Condition (RBC): At the wall i.e., at $y=0$, radiative boundary condition $-\frac{\partial T}{\partial y}=\Delta T x^m\left(T_w(x)-T_\infty\right)$ gives $m\to\infty$, hence the mixed thermal boundary condition reduces to $\theta(0)+\theta'(0)=0$.

V. SKIN-FRICTION AND WALL COUPLE STRESS

The non-dimensional skin friction C_f and wall couple stress M_w are given by

$$\frac{\mathrm{Re}^2}{Gr^{3/4}}C_f=2(1+K)f''(0) \quad \text{and}$$

$$\frac{\mathrm{Re}^2}{Gr}M_w=\left(1+\frac{K}{2}\right)g'(0).$$

VI. RESULTS AND DISCUSSION

Similarity analysis of free-convection boundary-layer flow about a vertical plate embedded in a micropolar fluid under mixed thermal boundary conditions in the presence of a transverse magnetic field using Lie group point transformations is studied. Using the Lie group point transformations on the governing equations result in a set of coupled second-order non-linear differential equations Eqs. (27) – (30) for $f(\eta)$, $g(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ along with the boundary conditions Eqs. (31) and (32). These equations (27) – (30) were solved numerically for the radiation boundary condition (RBC) case $(m\to\infty)$, by employing the local linearisation method (Motsa, 2013) which linearise the equations then apply the spectral collocation method for discretization and subsequent solution. We remark that the choice of linearisation method is influenced by the wall temperature conditions under investigation. It was observed that the local linearization method (LLM) is applicable for the variable wall temperature and variable wall heat flux condition. Applying the LLM on the radiation boundary condition gives only the trivial solution $\theta=0$ for temperature which is not meaningful physically. The non-trivial solution can be obtained by linearising the momentum and energy equations as a coupled pair of equations with unknown functions f and θ at any level of iteration. Below, we give the development of the iteration scheme used for the generation of results under the different wall temperature conditions. The derivation of the LLM scheme starts with writing the system of ODEs in compact operator form as

$$\Omega_1[F(\eta)]=0,\ \Omega_2[G(\eta)]=0,\quad \Omega_3[T(\eta)]=0,\ \Omega_4[P(\eta)]=0 \tag{33}$$

where Ω_1, Ω_2, Ω_3 and Ω_4 are non-linear operators that denote equation (27 – 30) respectively, and F, G, T, P are defined as

$$F=\left\{f,\frac{df}{d\eta},\frac{d^2f}{d\eta^2},\frac{d^3f}{d\eta^3}\right\},$$

$$G=\left\{g,\frac{dg}{d\eta},\frac{d^2g}{d\eta^2}\right\},$$

$$T=\left\{\theta,\frac{d\theta}{d\eta},\frac{d^2\theta}{d\eta^2}\right\},$$

$$P=\left\{\phi,\frac{d\phi}{d\eta},\frac{d^2\phi}{d\eta^2}\right\}$$

The assumption made in presenting the governing equations in the form (33) is that at any iteration level the dominant unknown functions are the ones corresponding to the term with the highest derivative in each equation. Linearising the first equation by applying the Taylor series expansion of Ω_1 about some previous approximation of the solution denoted by f_r, gives

$$(1+K)f'''_{r+1}+a_{11}^{(2)}f''_{r+1}+a_{11}^{(1)}f'_{r+1}+a_{11}^{(0)}f_{r+1}=R_1 \tag{34}$$

$$f_{r+1}=0,\ f'_{r+1}=0,\ f_{r+1}(\eta)=0 \quad \text{as } \eta\to\infty \tag{35}$$

where, $a_{11}^{(2)}=f_r$, $a_{11}^{(1)}=-2f'_r-M$, $a_{11}^{(0)}=f''_r$,
$R_1=-Kg'_r-\theta_r-B\phi_r+f_rf''_r-\left(f'_r\right)^2$.

In the above equation $r+1$ and r denote the current and previous iteration levels. It is worth noting that in view of the assumption that the solutions for g, θ and ϕ are known at the previous iteration level the solution for f can be obtained by solving (34) and (35) independently. The rest iteration schemes derived from the others equations are given by

$$\left(1+\frac{K}{2}\right)g''_{r+1}+a^{(1)}_{22}g'_{r+1}+a^{(0)}_{22}g_{r+1}=R_2,\ g_{r+1}=0,\ g_{r+1}(\eta)=0\ as\ \eta\to\infty \tag{36}$$

$$\frac{1}{\Pr}\theta''_{r+1}+a^{(1)}_{33}\theta'_{r+1}+a^{(0)}_{33}\theta_{r+1}=R_3,\ (1-m)\theta_{r+1}(0)-m\theta'_{r+1}(0)=1,\ \theta_{r+1}(\eta)=0\ as\ \eta\to\infty \tag{37}$$

$$\frac{1}{Sc}\phi''_{r+1}+a^{(1)}_{44}\phi'_{r+1}+a^{(0)}_{44}\phi_{r+1}=R_4,\phi_{r+1}(0)=1,\phi_{r+1}(\eta)=0\ \ as\ \ \eta\to\infty \tag{38}$$

The coefficients are defined as

$$\left.\begin{aligned}&a^{(1)}_{22}=f_{r+1},\quad a^{(0)}_{22}=-f'_{r+1}-2K,\quad a^{(1)}_{33}=f_{r+1},\quad a^{(0)}_{33}=-f'_{r+1},\\&a^{(1)}_{44}=a^{(1)}_{33},\quad a^{(0)}_{44}=a^{(0)}_{33},\quad R_2=2Kf''_{r+1},\quad R_3=R_4=0\end{aligned}\right\}$$

To solve the linearised system of equations (34 – 38) we use the standard spectral collocation method that transforms the continuous derivatives to the matrix vector products at N selected collocation points $\eta_j=\cos(\pi j/N)$, according to the definition

$$f'(\eta_j)=\sum_k^N D_{j,k}(\eta_k),\quad j-0,1,2,...,N. \tag{39}$$

The use of (39) leads to

$$\mathbf{F'}=\mathbf{DF},\ \mathbf{F''}=\mathbf{D^2F},\ \mathbf{F'''}=\mathbf{D^3F} \tag{40}$$

where $\mathbf{D}$ is the scaled differentiation matrix whose entries are defined in (Canuto et al., 1988) and $\mathbf{F}=[f(\eta_0),f(\eta_1),...,f(\eta_N)]^T$. Using equation (40) on the linearized equations (34 – 38) gives

$$\mathbf{A_1F}=\mathbf{R_1}\ ,\mathbf{A_2G}=\mathbf{R_2}\ ,\ \mathbf{A_3T}=\mathbf{R_3}\ ,\ \mathbf{A_4P}=\mathbf{R_4}$$

where

$$\left.\begin{aligned}&\mathbf{A_1}=(1+K)\mathbf{D^3}+a^{(2)}_{11}\mathbf{D^2}+a^{(1)}_{11}\mathbf{D}+a^{(0)}_{11},\qquad \mathbf{A_2}=(1+K/2)\mathbf{D^2}+a^{(1)}_{22}\mathbf{D}+a^{(0)}_{22}\\&\mathbf{A}_3=\frac{1}{\mathbf{Pr}}\mathbf{D^2}+a^{(1)}_{33}\mathbf{D}+a^{(0)}_{33},\quad \mathbf{A_4}=\frac{1}{Sc}\mathbf{D^2}+a^{(1)}_{44}\mathbf{D}+a^{(0)}_{44}\end{aligned}\right\}$$

To analyze the results for the present investigation under the radiation boundary condition (case(c)) $m\to\infty$ case, the values of $f''(0)$ and $\theta(0)$ were given in the Table 1 for $K=0=M=B$ and $Pr=10$. These values compared with the results given by Buyuk and Ece[7] for free convection in the absence of magnetic field were found to be in good agreement.

For the case (c), i.e., radiation boundary condition (RBC) case the Figs.1(a)–1(d) depict the variation of coupling number (K) on the profiles of velocity, microrotation, temperature and concentration with η. As for the consideration of the fluid model, the coupling number K characterised as a result of the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence, $K=\kappa/\mu$ signifies the coupling between the Newtonian and rotational viscosities. The effect of microstructure becomes significant for the increasing values of K while the individuality of the substructure is much less pronounced for the decreasing values of K. We can see that in the limiting case of $K\to 0$ i.e., $\kappa\to 0$, the micropolarity behaviour of the fluid vanishes and the fluid behaves as nonpolar fluid, and this leads to the case of viscous fluid. It is observed from Fig.1 (a) that as the value of K increases there is a asymptotical decrease in the velocity. From $K\to 0$ case i.e., the case of viscous to the non-viscous case there is a decrease in velocity. From Fig.1(b) it is evident that, at the wall there is a decrease in the component of microrotation. With the increment values of K the microrotation parameter increases in the ambient medium. As $\kappa\to 0$, i.e., $K\to 0$, the Eq. (3) becomes uncoupled with Eqs. (1) and (2) and they reduce to the equations of viscous fluid flow. It is very interesting to notice from Fig.1(c) that the temperature increases with the increasing values of coupling number. This is because of the decrease in fluid velocity, which causes to

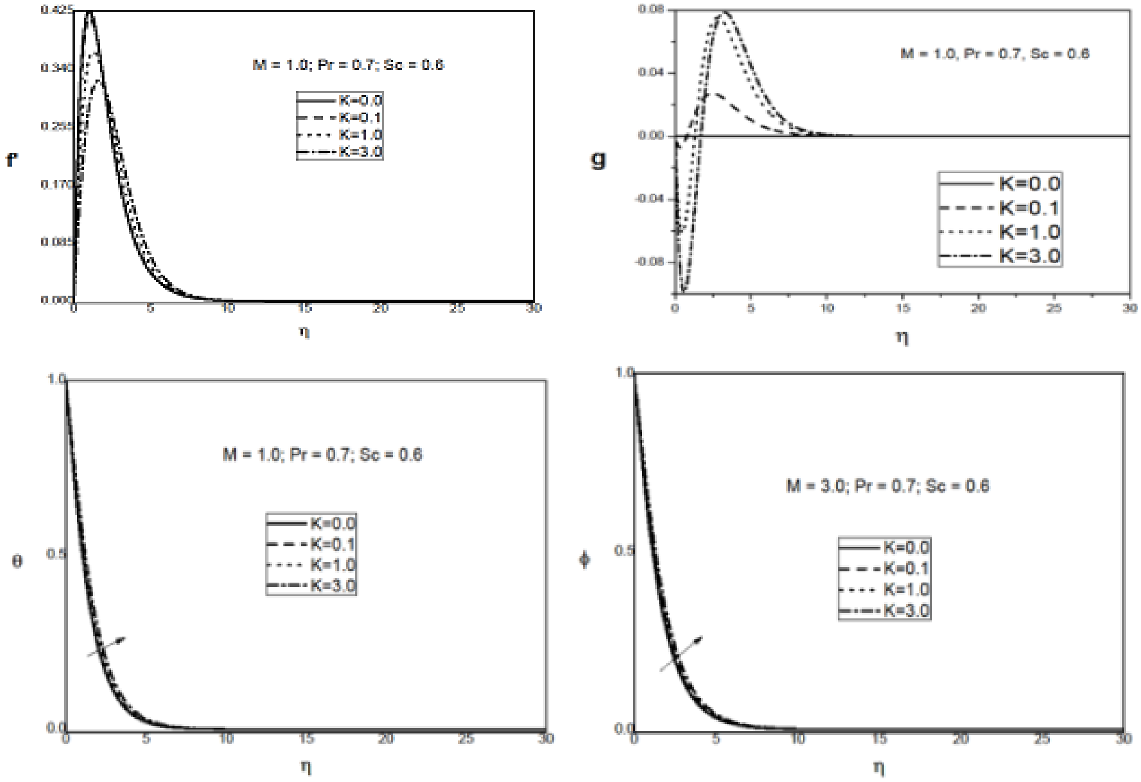
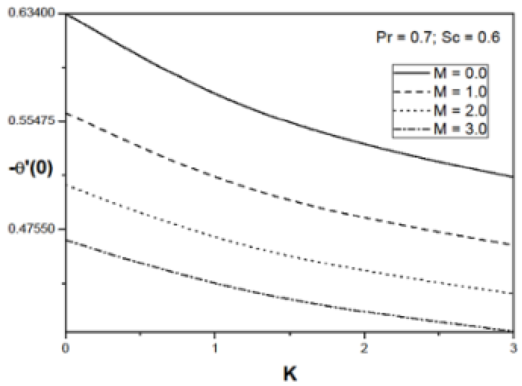
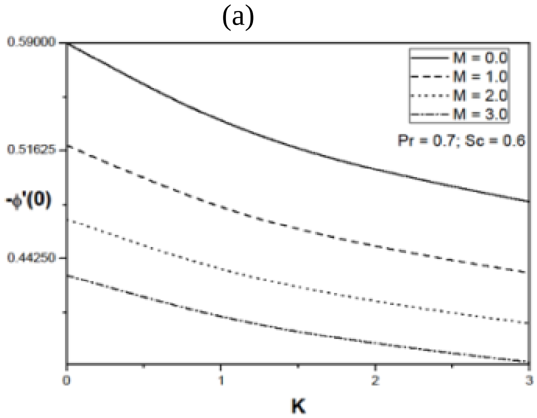


Figure 1: Velocity, microrotaton, temperature and concentration profiles for different values of K .

decrease the replacement of hot fluid chunks near the plate by cold fluid chunks. Same trend can be observed in Fig.1 (d) that from the case of non-Newtonian to the case of Newtonian concentration increases.



(a)



(b)

Figure 2: Profiles of heat and mass transfer rates as a function of K for different values of magnetic parameter (M).

Figs. 2(a) and 2(b) depict that the effect of coupling number (N) on heat transfer rate and mass transfer rate, for different M values. From the Figs. 2(a) and 2(b) it is observed that both the heat and mass transfer rates decrease for the increasing values of the magnetic parameter. This is because of the resistance increased between the fluid layers as a result of transverse magnetic field, which leads to Lorentz force. This leads to slow down the mechanism of replacement of hot fluid at the surface of the plate by the cold fluid chunks away from the plate. Hence the heat and mass transfer rates decrease as M increases.

Table 1

	[7]	**Present**
$f''(0)$	0.283227	0.283227
$\theta(0)$	0.496337	0.496337

Conflict of Interests

The authors confirm that there is no conflict of interest to declare for this publication.

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