



Computational Modelling of Gas-to-Solid Heat Transfer in an Adiabatic, Vertical Pipe

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Abstract: Gas-solid flows in vertical pipes are found in many industries for heat transfer applications. Some of them are chemical industries, food and process industries, pharmaceutical industries, etc. In the present paper, the two-fluid model (the Eulerian-Eulerian approach) of ANSYS FLUENT 15.0 is used to model the heat transfer in gas-solid flows in an adiabatic, vertical pipe. The variable gas properties with respect to temperature are considered in the current study. The computational results are well validated with the benchmark experimental data. The effect of particle diameter on heat transfer and pressure drop is studied. It is noticed that the gas temperature increases and the solid temperature decreases with increasing the particle diameter. Again, increasing the particle diameter increases the logarithmic mean temperature difference and pressure drop; however, it decreases the average gas-solid Nusselt number.

Keywords: gas-solid flows; heat transfer, numerical modeling, logarithmic mean temperature difference.

I. INTRODUCTION

Gas-solid flows in vertical pipes are found in many industries for heat transfer applications. Some of them are chemical industries, food and process industries, pharmaceutical industries, etc. Many authors (Farbar and Morley, 1957; Depew and Farbar, 1963; Farbar and Depew, 1963; Boothroyd and Haque, 1970; Wahi, 1977; Matsumoto et al., 1978; Kane and Pfeffer, 1985; Sorensen et al., 2001) experimentally studied thermo-hydrodynamics behaviour of gas-solid flows using heated walls in vertical pipes. Rajan et al. (2008) studied heat transfer during pneumatic conveying in an adiabatic, vertical pipe using gypsum particles. They studied the influences of different parameters such as solid feed rate, gas velocity, and particle diameter on heat transfer. Mokhtarifar et al. (2015) experimentally studied gas-solid heat transfer in adiabatic pipes and found that increasing the solid loading ratio (SLR) results in decreasing the gas-solid Nusselt number and solid temperature in dilute phase, and an opposite effect in dense phase.

Due to rapid advancement in computer science and information technology, numerical studies, as an alternative method, are also useful to obtaining the fluid dynamics results. There are two numerical approaches, i.e., the Eulerian-Lagrangian approach and

the Eulerian-Eulerian approach in gas-solid flow modeling. The Eulerian-Lagrangian approach treats the gas phase as continuous and the solid phase as discrete phase. However, the Eulerian-Eulerian approach treats both the phases as inter-penetrating continua. Many authors (Avila and Cervantes, 1995; Mansoori et al., 2002; Chagras et al., 2005; Saffar-Avval et al., 2007; Haim et al., 2007; Behzad et al., 2010; El-Beheri et al., 2011; Pishvar et al., 2014) used the Eulerian-Lagrangian approach and many other authors (Han et al., 1991; Boulet et al., 1999; Azizi et al., 2012; Patro, 2016) used the Eulerian-Eulerian approach to study the heat transfer from wall to gas-solid mixture in vertical pipes. Rajan et al. (2007) studied heat transfer between cold particles and hot air using plastic pellets of size 0.2 mm to 2 mm using the Eulerian-Eulerian approach in pneumatic conveying. They noticed the influence of particle diameter on gas-solid flow behavior. Bourloutski et al. (2002) compared the above two approaches of numerical modelling in gas-solid flows and concluded that the Lagrangian approach is limited to small SLRs (2-3), and the time required to reach the converged solution is 3-5 times more than the Eulerian-Eulerian approach. Using both numerically (the Eulerian-Lagrangian approach) and experimentally, El-Beheri et al. (2012) studied heat transfer during vertical pneumatic transport. They used hot gas with cold limestone particles and cold gas with hot

limestone particles in their study. They found that an increment in the gas and solid temperatures when the Reynolds number increases and the SLR decreases. They also noticed that the pressure drop increases in dilute regime and decreases in dense regime with hot particles and cold gas flow, and a reverse effect is noticed with cold particles and hot gas flow. Moreover, El-Behery et al. (2017) modeled gas-solid flows with heat transfer applying the steady state one-dimensional Eulerian-Eulerian approach, and it was found that the model is capable of modeling compressible gas-solid flows with heat transfer. They noticed an increment in the pressure drop when the SLR, solid diameter, and solid density increase.

There are limited published research works on heat transfer in gas-solid flows subjected to adiabatic walls, where the heat transfer happens from hot gas to cold

solids. Again, the thermal interactions between gas and solid phases are yet not well understood in these systems. Therefore, in this study, an attempt is taken to study the heat transfer (from hot air to cold sand particles) in a vertical pipe having an adiabatic wall. In the present study, **the two-fluid model** (the Eulerian-Eulerian approach) is employed with temperature variable gas properties.

II. MATHEMATICAL MODEL AND NUMERICAL PROCEDURE

The mathematical model is taken from **the two-fluid model** of ANSYS FLUENT 15.0. Neglecting mass transfer between the phases or source terms, continuity equation for gas phase and solid phase is

$$\frac{\partial}{\partial t}(\alpha_i \rho_i) + \nabla \cdot (\alpha_i \rho_i \mathbf{v}_i) = 0 \quad (1)$$

where 'i' is either gas or solid and $\sum \alpha_i = 1$.

Neglecting virtual mass force and external body forces, gas phase momentum equation is

$$\frac{\partial}{\partial t}(\alpha_g \rho_g \mathbf{v}_g) + \nabla \cdot (\alpha_g \rho_g \mathbf{v}_g \mathbf{v}_g) = -\alpha_g \nabla \bar{p} + \nabla \cdot \boldsymbol{\tau}_g + \alpha_g \rho_g \mathbf{g} + K_{sg}(\mathbf{v}_s - \mathbf{v}_g) \quad (2)$$

and solid phase momentum equation is

$$\frac{\partial}{\partial t}(\alpha_s \rho_s \mathbf{v}_s) + \nabla \cdot (\alpha_s \rho_s \mathbf{v}_s \mathbf{v}_s) = -\alpha_s \nabla \bar{p} - \nabla \bar{p}_s + \nabla \cdot \boldsymbol{\tau}_s + \alpha_s \rho_s \mathbf{g} + K_{gs}(\mathbf{v}_g - \mathbf{v}_s) \quad (3)$$

Neglecting radiation heat transfer, energy equation for gas phase is

$$\alpha_g \rho_g C_{pg} \left(\frac{\partial T_g}{\partial t} + \mathbf{v}_g \cdot \nabla T_g \right) = -\nabla \cdot \mathbf{q}_g + h_{gs}(T_s - T_g) \quad (4)$$

and the energy equation for solid phase is

$$\alpha_s \rho_s C_{ps} \left(\frac{\partial T_s}{\partial t} + \mathbf{v}_s \cdot \nabla T_s \right) = -\nabla \cdot \mathbf{q}_s - h_{gs}(T_s - T_g) \quad (5)$$

The various constitutive equations, which are required in the mathematical modelling, are presented in Table 1. Stress tensors ($\boldsymbol{\tau}$) are

$$\boldsymbol{\tau}_g = \alpha_g \mu_g (\nabla \mathbf{v}_g + \nabla \mathbf{v}_g^T) + \alpha_g \left(\lambda_g - \frac{2}{3} \mu_g \right) \nabla \cdot \mathbf{v}_g \mathbf{I} \quad (6)$$

$$\boldsymbol{\tau}_s = \alpha_s \mu_s (\nabla \mathbf{v}_s + \nabla \mathbf{v}_s^T) + \alpha_s \left(\lambda_s - \frac{2}{3} \mu_s \right) \nabla \cdot \mathbf{v}_s \mathbf{I} \quad (7)$$

$$\mu_s = \mu_{s,kin} + \mu_{s,coll} \quad (8)$$

$$\text{The heat transfer coefficient between phases } (h_{sg}) \text{ is, } h_{sg} = \frac{6k_g \alpha_s \alpha_g Nu_s}{d_s^2} \quad (9)$$

Properties of gas, i.e., density, dynamic viscosity, thermal conductivity and specific heat are defined with respect to temperature.

$$\rho_g = \frac{P}{R_A T_K} \quad (10)$$

where P is the operating pressure at atmospheric conditions. A temperature dependent piecewise-polynomial profile is used to define the normal dynamic viscosity of gas (μ_{gn}) (Fluent, 2003).

$$\mu_{gn}(T_K) = A - BT_K + CT_K^2 - DT_K^3 + ET_K^4 - FT_K^5 + GT_K^6 - HT_K^7 \quad (11)$$

where A, B, C, D, E, F, G and H are the coefficients, and $A=1161.482$, $B=2.368819$, $C=0.01485511$, $D=5.034909 \times 10^{-05}$, $E=9.928569 \times 10^{-08}$, $F=1.111097 \times 10^{-10}$, $G=6.540196 \times 10^{-14}$, $H=1.573588 \times 10^{-17}$.

Two separate user defined functions are provided to define the gas phase thermal conductivity (k_g) and specific heat at constant pressure (C_{pg}) as per Eq. 12 and Eq. 13, respectively (Dixon, 2007).

$$k_g = 0.02624 \left(\frac{T_k}{300} \right)^{0.8646} \quad (12)$$

$$C_{pg} = 1002.5 + 275 \times 10^{-6} (T_k - 200)^2 \quad (13)$$

Table 1. Different models used in constitutive equations

Terms	Models used
Granular bulk viscosity (λ_s)	Lun et al. (1984)
Granular viscosity (μ_s)	Syamlal et al. (1993)
Solid pressure	Lun et al. (1984)
Radial distribution function	Lun et al. (1984)
Turbulence	Standard $k - \varepsilon$ turbulence model (Launder and Spalding, 1974)
Granular temperature	PDE granular temperature model (Ding and Gidaspow, 1990)
Drag force	When $\alpha_g > 0.8$, Wen and Yu (1966) But when $\alpha_g \leq 0.8$, Ergun (1952)
Particle Nusselt number	Gunn (1978)

A fully developed velocity profile is used for gas phase boundary condition at the inlet. However, a uniform velocity profile (equal to gas velocity) is used for solid phase at the inlet. At the outlet, the outflow boundary condition is used for both phases. A no-slip wall boundary condition is used for gas, and a partial-slip wall boundary condition as given by Johnson and Jackson (1987) with a specular coefficient 0.05 is used for solid. The restitution coefficients for particle-particle and particle-wall collisions are 0.9 and 0.95, respectively. The wall is specified at the adiabatic condition.

Due analytical complexity to solve the nonlinear and coupled equations, the numerical approach is used. The commercial software package ANSYS 15.0 is used for geometric modeling (a vertical pipe of internal diameter 0.058 m and length 6 m), meshing, and transient simulations. The simulations are carried out using **the two-fluid model** of ANSYS FLUENT 15.0. In **the two-fluid model**, both phases are considered as the inter-penetrating continua. **The two-fluid model** is the most suitable model for gas-solid flows (Sundaresan, 2000). For the pressure and velocity coupling, the PC-SIMPLE (Phase Coupled Semi-Implicit Method for Pressure Linked Equations) algorithm is used. For the momentum and energy equations, an upwind scheme of second-order is used. For the volume fraction equations, the QUICK scheme is used. For the turbulent kinetic energy, turbulent energy dissipation rate, and granular temperature equations, a first-order upwind scheme is considered. The time-step size is 0.001s. A convergence criterion of 10^{-3} is used for all.

III. DATA REDUCTION

The calculation of average gas-solid Nusselt number is explained below (Mokhtarifar et al., 2015). From the energy balance

$$\dot{m}_g C_{pg} (T_{g,in} - T_{g,out}) + \dot{m}_s C_{ps} (T_{s,in} - T_{s,out}) = 0 \quad (14)$$

The overall heat transfer coefficient (U) is

$$\dot{m}_g C_{pg} (T_{g,in} - T_{g,out}) = UA_s (LMTD) \quad (15)$$

The heat transfer area is calculated as

$$A_s = 6M_s / (\rho_s \cdot d_s) \quad (16)$$

Here, M_s is the solid holdup and is calculated as

$$M_s = (\dot{m}_s \cdot \Delta z) / v \quad (17)$$

The Logarithmic Mean Temperature Difference (LMTD) is calculated as

$$LMTD = (\Delta T_{in} - \Delta T_{out}) / (\ln(\Delta T_{in} / \Delta T_{out})) \quad (18)$$

$$\Delta T_{in} = T_{g,in} - T_{s,in} \quad (19)$$

$$\Delta T_{out} = T_{g,out} - T_{s,out} \quad (20)$$

The local gas-solid Nusselt number (Nu_l) is calculated as

$$Nu_l = UD / k_g \quad (21)$$

The average gas-solid Nusselt (Nu_{avg}) number is calculated as

$$Nu_{avg} = \int_0^L Nu_l \cdot dz / L \quad (22)$$

IV. RESULTS AND DISCUSSION

4.1. Validation

Before doing the validation, a grid test is conducted using three grid sizes such as 114000 cells, 252000 cells and 504000 cells, keeping all other parameters constant. It is observed that, by changing the grid size from 252000 to 504000 cells, the gas and solid temperatures are negligibly affected. Hence, the grid of 252000 cells is used in the simulations to save the computational time.

The present computational results for average SLR and average gas-solid Nusselt number are compared with the experimental results of Mokhtarifar et al. (2015) and are plotted in **Fig. 1a** and **Fig. 1b**, respectively. Air is used as the gas phase, and sand (density 1500 kg/m^3 , specific heat 800 J/kgK , thermal conductivity 0.8 W/mK) is used as the solid phase. The mean gas velocity is 18.5 m/s , and sand particles are of 253 micron in size. The inlet air temperature is 443.15 K , and the inlet solid temperature is 308.15 K . It is noticed from both the figures that the present numerical results agree well with the experimental results of Mokhtarifar et al. (2015). The present numerical results show a maximum deviation of 3% for the average SLR and a maximum deviation of 6% for the average gas-solid Nusselt number with the benchmark experimental data.

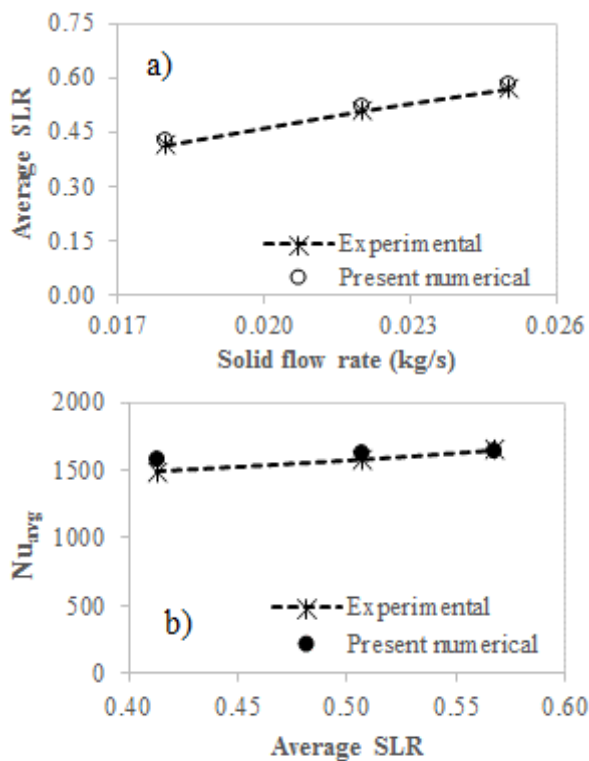


Figure 1: Comparison of numerical results with experimental results of Mokhtarifar et al. (2015) for average SLR and average gas-solid Nusselt number

4.2. Effect of particle diameter

Now the parametric studies are conducted using the simulation, considering different particle sizes such as 100 micron , 200 micron , and 300 micron . The mean gas velocity is 18 m/s (inlet gas Reynolds number 33850), and the SLR at the inlet is 0.7 . The effects of particle diameter on gas and solid temperatures, LMTD, average gas-solid Nusselt number, and pressure drop are studied.

The particle diameter versus gas and solid temperatures at a distance 0.5 m from the inlet is plotted in **Fig. 2**. It is seen from **Fig. 2** that the gas temperature increases and the solid temperature decreases with an increase in the particle diameter. This is due to decrease in the **particle residence time**. **Particle residence time** decreases with an increase in the particle diameter because of less number of particles.

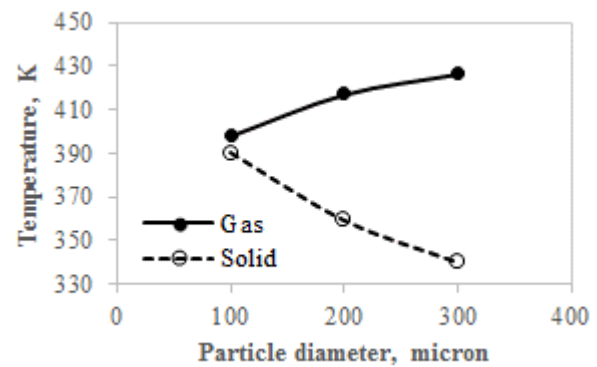
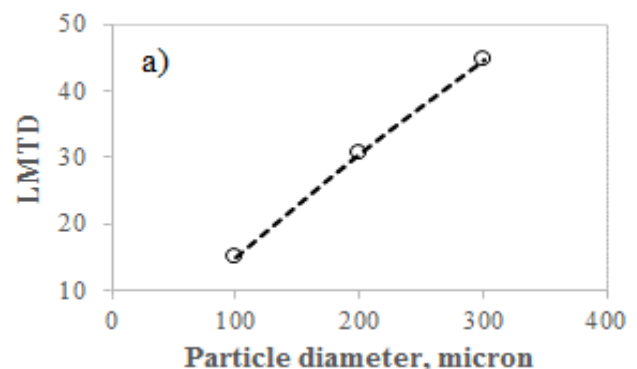


Figure 2: Particle diameter versus gas and solid temperatures

The effect of particle diameter on LMTD is plotted in **Fig. 3a**. It is seen from **Fig. 3a** that the LMTD increases with increasing the particle diameter. By increasing the particle diameter, the gas temperature increases and the solid temperature decreases. Therefore, the **temperature difference** between gas and solid increases, which increases the LMTD.



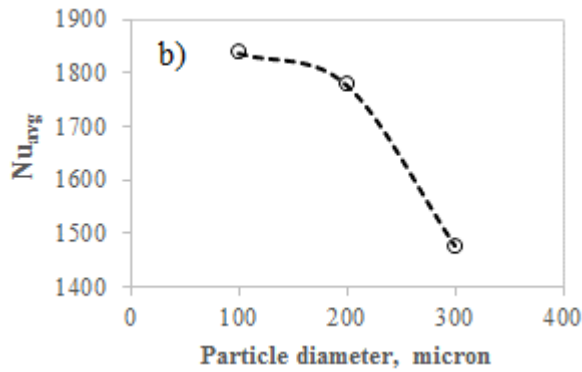


Figure 3: Particle diameter versus LMTD and average gas-solid Nusselt number

The effect of particle diameter on average gas-solid Nusselt number is plotted in **Fig. 3b**. It is seen from **Fig. 3b** that the gas-solid Nusselt number decreases with an increase in the particle diameter. The reason is due to **turbulence suppression** by the solid particles with increasing the particle diameter.

The effect of particle diameter on pressure drop is plotted in **Fig. 4**. It is seen from **Fig. 4** that the pressure drop increases with increasing the particle diameter. Increasing the particle diameter increases the **slip velocity** between gas and solid, and the increased **slip velocity** increases the drag force. Therefore, the pressure drop increases with increasing the particle diameter.

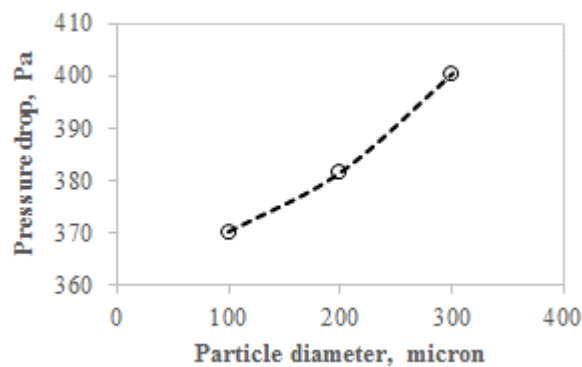


Figure 4: Particle diameter versus pressure drop

V. CONCLUSION

In the present paper, computational modeling of gas-solid flows through a three-dimensional vertical pipe having an adiabatic wall is conducted, considering the **two-fluid model** (the Eulerian-Eulerian approach) of ANSYS FLUENT 15.0. It is known that the gas properties vary with temperature and affect the fluid dynamics study. Therefore, the properties of gas are defined as per the temperature variation along the length of the pipe. First, the computational results are validated with the benchmark experimental data. The computational results show a maximum deviation of 6% with the benchmark experimental data for gas-solid Nusselt number. Then, the parametric studies are

conducted using the particle diameter ranging from 100 micron to 300 micron at the mean gas velocity 18 m/s and inlet SLR 0.7. It is noticed that the gas temperature increases and the solid temperature decreases with increasing the particle diameter. Increasing the particle diameter increases the LMTD and pressure drop; however, it decreases the average gas-solid Nusselt number.

Future scope of the paper

The study can be extended to other geometries such as inclined pipes, rectangular channels and square channels having adiabatic walls.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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Nomenclature

A_s	heat transfer area of particles (m^2)
C_p	specific heat at constant pressure (J/kgK)
d_s	particle diameter (m)
D	diameter of the pipe (m)
g	acceleration due to gravity (m/s^2)
h_{gs}	gas-solid heat transfer coefficient (W/m^2K)
I	unit tensor
k_g	thermal conductivity of gas (W/mK)
K_{gs}, K_{sg}	gas-solid momentum exchange coefficient (kg/m^3s)
L	pipe length (m)
\dot{m}	mass flow rate (kg/s)
M_s	solid holdup (kg)
Nu	gas-solid Nusselt number
Nu_s	particle Nusselt number
P	operating pressure (Pa)
\bar{p}	mean pressure (Pa)
R_A	specific gas constant (J/kgK)
T	temperature (K)
T_K	temperature of gas (K)
ΔT	temperature difference between gas and solid (K)
q	heat flux (W/m^2)
v	mean velocity (m/s)
U	overall heat transfer coefficient (W/m^2K)
z	axial distance (m)
Δz	distance from particle feeding point (m)
LMTD	logarithmic mean temperature difference
SLR	solid loading ratio
α	volume fraction
λ	bulk viscosity (kg/ms)
μ	shear viscosity (kg/ms)

μ_{gn}	normal dynamic viscosity of gas (kg/ms)
ρ	density (kg/m ³)
τ	stress-strain tensor (kg/ms ²)

Subscripts

i	gas or solid
avg	average
g	gas
in	inlet
l	local
out	outlet
s	solid

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