International Journal of Advanced Trends in Computer Applications
www.ijatca.com

Adomian Decomposition Method for Nonlinear Thermal Convection Flow of a Nanofluid Between Vertical Channel with Radiation Effect

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Abstract: *The main objective of this work is to investigate the effect of thermal radiation on nonlinear mixed convective flow of a nanofluid between two vertical parallel plates. The governing equations are transformed into a system of nonlinear differential equations using suitable non-dimensional transformations. The resulting differential equations are solved using an efficient Adomian decomposition method. The effects of the physical parameters on the developments of flow, temperature, nanoparticle volume fraction, heat transfer and nanoparticle mass transfer characteristics between parallel plates are given and the salient features are discussed. The combined effects of change in different physical parameters on different profiles are displayed graphically and quantitatively.*

Keywords: Adomian Decomposition Method, Thermal Radiation, Nanofluid, Nonlinear Thermal Convection.

I. INTRODUCTION

One of the main obstacles in the enhancement of heat transfer in various engineering systems is the low thermal conductivity of regular fluids like oil, air, water, and ethylene glycol mixture etc. A very popular and excellent method of improving the heat transfer in different kind of thermal systems is to increase the thermal conductivity of the base fluids by the suspension of nanoparticles in it because nanoparticles have very high thermal conductivities comparatively base fluids (Sarkar et al., 2015). The fluids with suspended nanoparticles (a kind of special colloids) were termed as nanofluids by Choi et al. (1995) for the very first time. From the review of the open literature, it is found that a lot of attention has been paid to the fully developed mixed convection flow of nanofluid in vertical channels (Fakour et al., 2014; Srinivasacharya and Shafeeurrahman, 2017).

In many thermal engineering areas, due to the wide range of applications in solar energy and cooling system design for electronic devices, the vertical channel is found to be a very frequently used configuration. The effect of radiation on convective flow of nanofluid together with heat and mass transfer from bodies of different geometries under various physical conditions have so many applications involving high temperatures such as gas turbines missiles, satellites, nuclear power plant, aircraft, and space vehicles etc. Due to this importance, many researchers have studied the effect of thermal radiation in different fluids with different geometries (Das et al., 2016; Srinivasacharya and Shafeeurrahman, 2017; Singh and Kumar 2016; Mandal and Mukhopadhyay 2018).

In some cases, the temperature dependence relation with density in the buoyancy force term is nonlinear and then the radiation effect is of immense importance. The non-linear variation in temperature-dependent density relation is termed as nonlinear thermal convection. In the fields of solar collectors, combustion and areas of reactor safety, this physical concept is more useful and gives motivation for many researchers to analyze nonlinear convection in boundary layer analysis (Hayat et al., 2015).
In the present study, the main objective is to investigate the significance of nonlinear temperature dependent density relation on the mixed convective flow of a nanofluid through the vertical channel with radiation effect. In order to analyze all the important essential features of the present problem considered, the resulting nonlinear governing equations are solved using the very effective semi-analytical method, known as the Adomian decomposition method (Adomian

1988; 1994).

II. FORMULATION OF THE PROBLEM

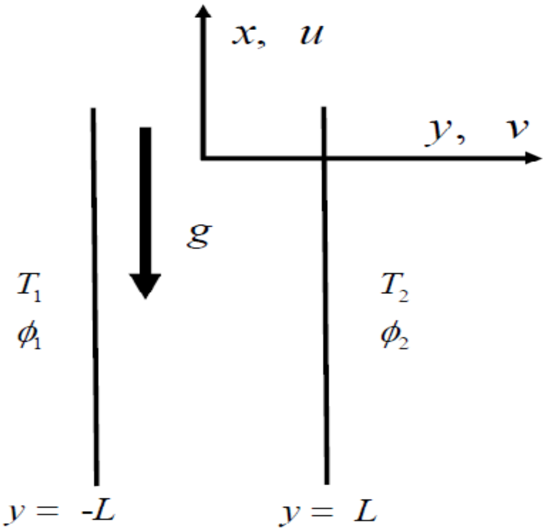

Figure 1: Physical model and coordinate system

Consider a fully developed flow of a nanofluid between vertical parallel plates distance $2L$ apart. We select such a coordinate system in which *x*-axis is taken in vertically upward direction, the *y*-axis is taken perpendicular to the plates and the two plates, in the direction of *x* and *z* are extended infinitely. The plates $y=-L$ (left plate) and $y=L$ (right plate) are maintained at the constant temperatures T_1 and T_2 respectively and nanoparticle volume fraction ϕ_1 and ϕ_2 respectively. Further, each fluid property is assumed as constant except density variations in the buoyancy force term. Here, the radiation effect is also considered. The buoyancy forces together with uniform pressure gradient (in the *x*-direction) causes the mixed convection flow. It is also assumed that the fluid velocity vector $\bar{\mathrm{v}}(u,v)$ is parallel to the x-axis so that *u* does not vanish and the transverse velocity *v* is zero because the flow is fully developed.

With the above-mentioned assumptions, the governing equations for non-linear convective flow of a nanofluid are given by

$$v\frac{du}{dy}=\upsilon\frac{d^2u}{dy^2}+g(1-\phi_0)\left[\beta_1\left(T-T_0\right)+\beta_2\left(T-T_0\right)^2\right]-g\frac{(\rho_P-\rho_{f_\infty})}{\rho_{f_\infty}}(\phi-\phi_0)-\frac{1}{\rho_{f_\infty}}\frac{dp}{dx} \tag{1}$$

$$v\frac{dT}{dy}=\alpha_m\frac{d^2T}{dy^2}+\tau\left[D_B\frac{d\phi}{dy}\frac{dT}{dy}+\frac{D_T}{T_0}\left(\frac{dT}{dy}\right)^2\right]+\frac{16\sigma^*T_0^3}{3\rho C_P K^*}\frac{d^2T}{dy^2} \tag{2}$$

$$v\frac{d\phi}{dy}=D_B\frac{d^2\phi}{dy^2}+\frac{D_T}{T_0}\frac{d^2T}{dy^2} \tag{3}$$

where velocity components along the *x* and *y* directions are *u* and *v* respectively, kinematic viscosity is denoted by υ, *g* represents gravity acceleration, *T* indicates temperature of the fluid, β_1 and β_2 are the first and second order thermal expansions, *ρ*, ρ_P and ρ_{f_∞} are densities of the fluid, nanoparticles, and base fluid respectively, p is the pressure, α_m is the thermal diffusivity, $\tau=(\rho c)_f/(\rho c)_p$ is the ratio of heat capacity of the base fluid and nanoparticles, Brownian Motion coefficient and thermophoresis diffusion coefficient are indicated as D_B and D_T respectively, C_p is the specific heat, σ^* is the Stefan-Boltzmann constant, K^* is the mean absorption coefficient and ϕ is the nanoparticle volume fraction of the fluid.

The boundary conditions are given by

$$u=0,\ T=T_1,\ \phi=\phi_1\quad\text{at}\quad y=-L \tag{4a}$$

$$u=0,\ T=T_1,\ \phi=\phi_1\quad\text{at}\quad y=+L \tag{4b}$$

Introducing the following non-dimensional transformations

$$Y=\frac{y}{L},U=\frac{u}{U_0},\theta=\frac{\mathrm{T}-\mathrm{T}_0}{\mathrm{T}_2-\mathrm{T}_0},\gamma=\frac{\phi-\phi_0}{\phi_2-\phi_0} \tag{5}$$

Substitute Eq. (5) in Eqs. (1) – (3), we obtain this nonlinear ODE system

$$U''-RU'+\frac{Gr}{\mathrm{Re}}[\theta(1+\lambda_1\theta)]-Nr.\gamma+A=0 \tag{6}$$

$$\frac{1}{\mathrm{Pr}}\left(1+\frac{4}{3}Rd\right)\theta''-R.\theta'+Nb.\frac{d\gamma}{dY}\frac{d\theta}{dY}+Nt.\left(\frac{d\theta}{dY}\right)^2=0 \tag{7}$$

$$\frac{1}{Le}\gamma''-R.\gamma'+\frac{1}{Le}\frac{Nt}{Nb}\theta''=0 \tag{8}$$

where primes denote differentiation with respect to Y, $\mathrm{Re}=\frac{v_0 L}{\upsilon}$ is the Reynolds number, R denotes suction/injection parameter, $\mathrm{Pr}=\frac{\upsilon}{\alpha_m}$, $Gr=\frac{(1-\phi_0)g\beta_1 L^3}{\upsilon^2}(T_2-T_1)$, $Le=\frac{\upsilon}{D_B}$ denote Prandtl number, Grashof number and Lewis number respectively, $A=-\frac{L^2}{\mu U_0}\frac{dp}{dx}$ denotes constant pressure gradient, λ_1 is the non-linear density temperature parameter $Nr=\frac{g(\rho_P-\rho_{f_\infty})(\phi_2-\phi_0)d^2}{\mu U_0}$ is the nanoparticle buoyancy ratio, $Rd=\frac{4\sigma^* T_0^3}{\rho\alpha_m C_P K^*}$, $Nb=\frac{\tau D_B(\phi_2-\phi_0)}{\upsilon}$ and $Nt=\frac{\tau D_T(T_2-T_0)}{T_0\upsilon}$ denote the thermal radiation parameter, the Brownian motion parameter, and the thermophoresis parameter respectively.

The boundary conditions can be written in terms of U, θ, ϕ as

$$\mathrm{U}=0,\ \theta=r_{\mathrm{T}},\ \gamma=r_\phi \quad \text{at} \quad Y=-1 \tag{9a}$$

$$\mathrm{U}=0,\ \theta=1,\ \gamma=1 \quad \text{at} \quad Y=1 \tag{9b}$$

where r_{T} and r_ϕ are the wall temperature and nanoparticle concentration parameters.

The non-dimensional heat and nanoparticle mass transfer rates on the left and right plates are denoted by Nu_1, Nu_2, Sh_1 and Sh_2 respectively and those are defined as

$$Nu_1=\theta'(-1),\ Nu_2=\theta'(1) \text{ and } Sh_1=\gamma'(-1),\ Sh_2=\gamma'(1) \tag{10}$$

III. ANALYSIS AND DISCUSSION

The solutions for $U(Y)$, $\theta(Y)$ and $\gamma(Y)$ have been computed with a novel semi-analytical method known as ***Adomian Decomposition Method*** (ADM) [9,10]. The graphical presentation of the obtained results (shown in Figs. 2 to 5) is used to study the influence of pertinent parameters, like thermal radiation parameter (Rd), thermophoresis parameter (Nt), and Prandtl number (Pr), etc. Moreover, the computations are carried out by taking Nr *=1.0*, Nb =0.3, λ_1 =0.5, Pr *=1.0*, Le *=10.0*, $r_{\mathrm{T}}=0.1$, $r_\phi=1.0$, *Gr = 10.0*, *Re = 2.0*, *R = 2.0* and *A =1.0* (unless otherwise mentioned).

Figure 1 shows the effect of thermal radiation parameter (Rd) on U (*Y*), $\theta(Y)$ and $\gamma(Y)$ respectively. It is visible that the velocity *U*(*Y*) increases with thermal radiation parameter increment. The velocity *U(Y)* is maximum in the middle of the channel and is reduced near the boundary of the channel and satisfies the boundary conditions. It is also visible that $\theta(Y)$ increases with *Rd* increment. The temperature of the fluid increases as we move from the left plate to the right plate but the nanoparticle volume fraction $\gamma(Y)$ decreases with *Rd* increment. Here, $\gamma(Y)$ increases as we move towards the middle of the channel from the left plate and it becomes constant in the middle and then again increases near the right plate.

Figures 2 represents the effect of the thermophoresis parameter (Nt) on velocity components *U*(*Y*), temperature θ(*Y*) and nanoparticle volume fraction $\gamma(Y)$ respectively. It is visible that *U*(*Y*) decreases with thermophoresis parameter (Nt) increment. But θ(*Y*) escalates with *Nt* increment. It is because of the thermophoresis force due to which particles from hot zone move to cold zone. Also, $\gamma(Y)$ increases very effectively with an increase in the thermophoresis parameter Nt. For a small value of Nt, $\gamma(Y)$ increases towards the middle of the channel from the left plate and becomes constant in the middle and later again increases towards the right plate. But for a large value of Nt (say 1.0), the nanoparticle volume fraction of fluid increases very rapidly and becomes maximum at the middle of the channel and then it starts decreasing and finally satisfies boundary conditions.

Figures 3 represents the effect of the Prandtl number (Pr) on velocity components *U*(*Y*), temperature θ(*Y*) and nanoparticle volume fraction $\gamma(Y)$ respectively. It is found that *U*(*Y*) decreases with Pr increment. But, the temperature of the fluid θ(*Y*) shows the opposite behavior. The nanoparticle volume fraction $\gamma(Y)$ also shows a similarity with θ(*Y*).

Figures 4 shows the influence of different parameters namely nonlinear density temperature λ_1, Brownian motion parameter Nb, Grashof number Gr , and Reynolds number *Re* on the velocity profiles of the nanofluid. When we increase Nb, consequently, there is an enhancement in the temperature due to nanoparticle diffusion into the fluid. On increasing the value of λ_1 the non-dimensional velocity also increases. This velocity is maximum at the middle of the channel and satisfies the boundary conditions. The similar variation is observed in the case of Nb and it is important to note that, for higher values of Nb the change in corresponding velocities is very less. As increase the value of Grashof number, the velocity also gets increased in a uniform manner. It should be noted that with the increase in the value of *Gr* the corresponding maximum velocity is shifted towards the middle of the channel, i.e. for a smaller value of *Gr* the maximum velocity is near the left plate. The variation of velocity $U(Y)$ with a Reynolds number in this channel is very uniform. As increase, the value of *Re* the velocity decreases. The velocity is maximum at the middle of the channel in this case too and satisfies boundary conditions.

For both the plates, variation in heat and mass transfer rates with effects of Nt, Nb, λ_1 and Rd are given in Table-1. For the fixed values of $Nb = 0.3,\ \lambda_1 = 0.5$ and $Rd = 0.5$, the heat transfer rate decreases with the increase of Nt at both the plates. But the nanoparticle mass transfer rate is not similar to this one. It increases on the left plate but decreases on the right plate. It can be seen from the Table-1 that when we fix the values $Nt = 0.2, \lambda_1 = 0.5$ and $Rd = 0.5$, the heat transfer rate decreases with the increase of Nb at both the plates. But the nanoparticle mass transfer rate is different from this one. It decreases on the left plate but increases on the right one. From the Table-1 we can also observe that when we fix $Nt = 0.2, Nb = 0.3$ and $Rd = 0.5$, the heat transfer rate and nanoparticle mass transfer rate are constant on both the plates. Similarly, from the Table-1 it is visible that when we fix the values $Nt = 0.2, Nb = 0.3$ and $\lambda_1 = 0.5$, the heat transfer rate decreases with an increase of Rd at both the plates. But the nanoparticle mass transfer rate shows a different nature. It decreases on the left plate but increases on the right plate.

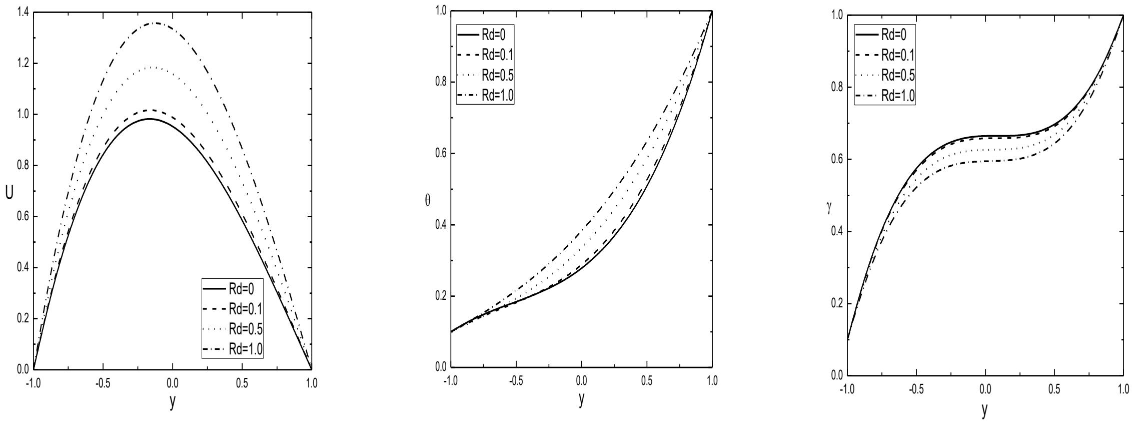

Figure 1: Effect of Rd on (a) $U(Y)$, (b) $\theta(Y)$ and (c) $\gamma(Y)$ profiles.

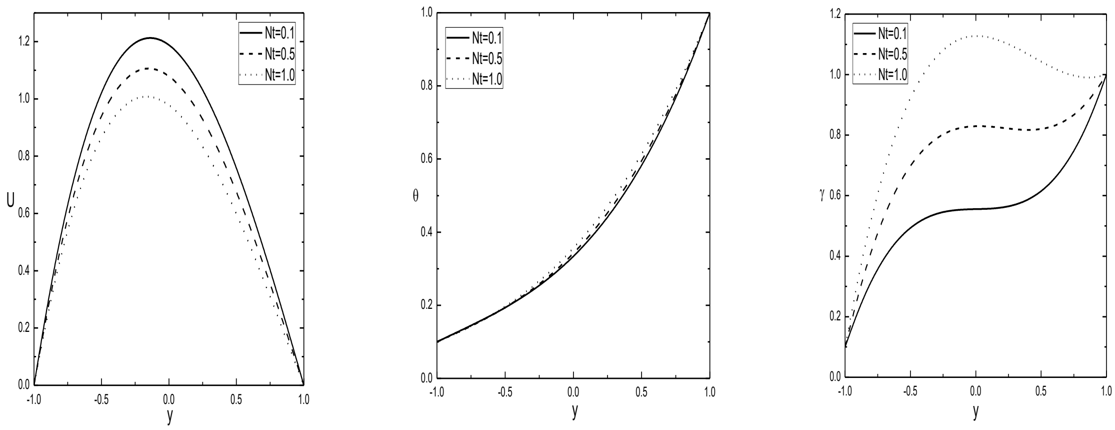

Figure 2: Effect of Nt on (a) $U(Y)$, (b) $\theta(Y)$ and (c) $\gamma(Y)$ profiles.

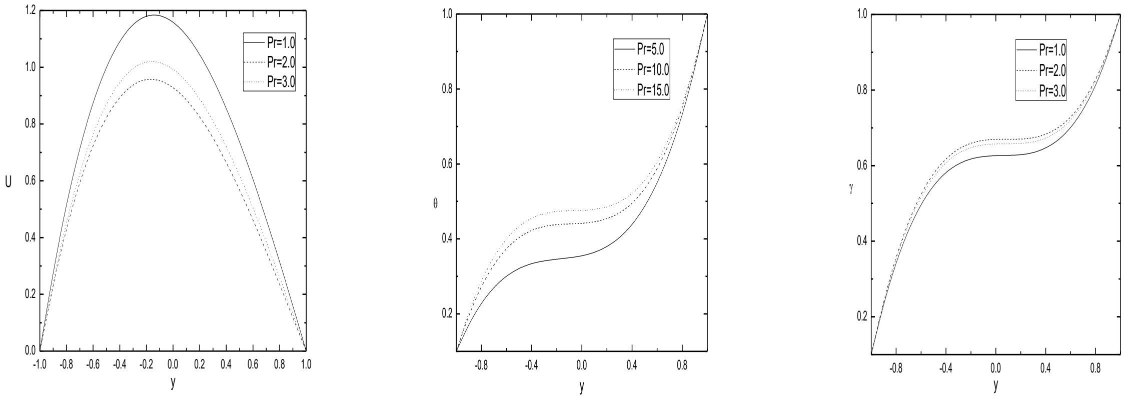

Figure 3: Effect of Pr on (a) $U(Y)$, (b) $\theta(Y)$ and (c) $\gamma(Y)$ profiles

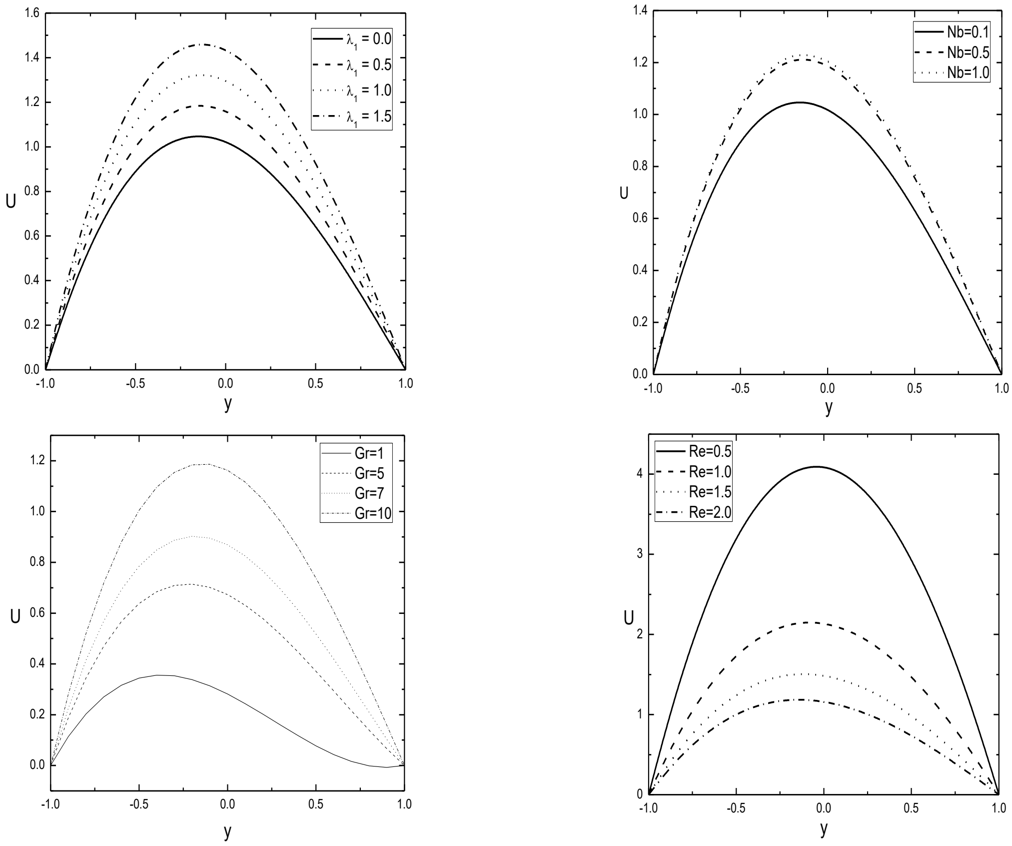

Figure 4: Effect of variation in (a) λ_1, (b) Nb, (c) Gr and (d) Re on velocity profiles

IV. CONCLUSIONS

The main outcomes of this mixed convective flow study of nanofluid between vertical parallel plates in the presence of nonlinear thermal convection and radiation effects can be summarized as:

- With thermal radiation parameter increment, the velocity and temperature of the nanofluid increase but opposite with the nanoparticle volume fraction of the fluid. The heat transfer rate decreases with an increase of Rd at both plates but the nanoparticle mass transfer rate shows a different nature. It decreases on the left plate but increases on the right plate.

- As the thermophoresis parameter increases the velocity of the nanofluid decreases and the temperature and nanoparticle volume fraction of the nanofluid increases. The heat transfer rate decreases with increment in Nt at both the plates. But for the nanoparticle mass transfer rate, we observe that it increases on the left plate but decreases on the right plate.

- There is a decrement in velocity but the increment in temperature of fluid when we increase the Prandtl number. The nanoparticle volume fraction also increases with Prandtl number like the temperature of the fluid.

Table-1: Nusselt Number and nanoparticle Sherwood Number variation with Nt, Nb, λ_1 and Rd.

Nt	Nb	λ_1	Rd	Nu_1	Nu_2	Sh_1	Sh_2
0.1	0.3	0.5	0.5	0.1806	1.04694	1.34808	1.32532
0.5	0.3	0.5	0.5	0.1649	0.99776	1.89774	0.77566
1	0.3	0.5	0.5	0.1512	0.92629	2.49552	0.17788
0.2	0.1	0.5	0.5	0.17901	1.0364	2.06109	0.61231
0.2	0.5	0.5	0.5	0.17382	1.03415	1.37576	1.29764
0.2	1	0.5	0.5	0.16725	1.03131	1.2901	1.3833
0.2	0.3	0	0.5	0.17642	1.03528	1.48998	1.18342
0.2	0.3	0.5	0.5	0.17642	1.03528	1.48998	1.18342
0.2	0.3	1	0.5	0.17642	1.03528	1.48998	1.18342
0.2	0.3	0.5	0	0.24388	1.33369	1.56696	1.10643
0.2	0.3	0.5	0.1	0.21106	1.26056	1.55353	1.11987
0.2	0.3	0.5	0.5	0.17642	1.03528	1.48998	1.18342

In the presence of thermal radiation effect, the nanoparticle mass transfer rate is showing the opposite trend on the two plates, but the heat transfer rate is not showing any nature of this kind and there is uniformity.

Conflict of Interest

Both the authors have equal contribution in this work and it is declared that there is no conflict of interest for this publication.

Acknowledgment

The authors express their sincere gratitude to the referees for their valuable suggestions towards the improvement of the paper.

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