



# Effects of Radiation and Radiation Absorption on Unsteady MHD Flow Past a Vertical Porous Flat Plate in a Rotating System with Chemical Reaction in a Nanofluid

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**Abstract:** In this paper unsteady MHD Ag-nanofluid based boundary flow in a rotating system by means of chemical reaction as well as radiation in presence of heat absorption has been investigated. The partial differential equations of system were solved using perturbation technique. The influence of diverse physical parameters on concentration, velocity as well as temperature fields is illustrated graphically. In this examination it was established that the Nano fluid velocity rises with the enhancement of porous medium and thermal radiation. However, concentration, velocity as well as temperature declined with amplification in chemical reaction.

**Keywords:** Radiation, Ag-water Nanofluid, sphere shape particles, heat transfer, porous.

## I. INTRODUCTION

Choi (1995), was the first one who reported regarding the fluids, renowned as Nano fluids wherein particles of Nano scale is suspended in the pedestal fluid. Aluminium, copper, iron as well as titanium or else their oxides are a few of the common Nano particles which are used widely. Owing to high thermal conductivity, Nano fluids have several useful applications and they are used on a large scale in manufacturing industries, fuel cells, solid-state lighting, and as coolants in auto mobiles etc. Numerous methods have been developed to improve the thermal conductivity of these fluids by suspending Nano particles in liquids. Khanafer et al. (2003) analyzed buoyancy driven heat transfer enhancement in a two dimensional enclosure utilizing Nano fluids. Ishak(2010) examined similarity and analytical solution for fluid flow as well as heat transfer in excess of a permeable surface by means of convective boundary condition. Khairy Zaimi et al..(2013) reported the flow

as well as heat transfer over a shrinking sheet in a nanofluid with suction at the boundary. Kim et al. (2004) & Hamad et al. (2011) examined unsteady MHD free convective flow past a non-parallel permeable flat plate in a rotating frame of reference by means of stable heat source in a nanofluid. The fluid flow over a rotating as well as stretching disk and the flow between two stretching disks were studied by Fang et al (2007, 2008). Prasad et al. (2010), Turkyilmazoglu (2012) discussed on MHD Nano fluid flows over a stretching sheet in dissimilar environments. Fang and Hua (2012), Rashidi et al. (2013) investigated the fluid flows over stretchable rotating disks. Bachok et al.(2011) used Keller-Box technique for steady Nano fluid flow over a porous rotating disk.

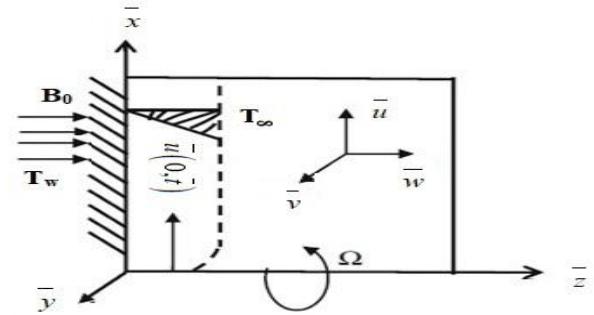
The main aim is to examine the unsteady MHD Ag-nanofluid based boundary flow in a rotating system by means of chemical reaction as well as radiation in existence of heat absorption. The partial differential equations of system were solved using perturbation

technique. The influences of diverse physical parameters on concentration, velocity as well as temperature fields are illustrated graphically.

## II. MATHEMATICAL FORMULATION AND SOLUTION

Unsteady MHD 3D natural convective nanofluid flows which passes through a semi-infinite vertical permeable plate which is grey emitting or absorbing but not scattering medium in the influence of heat absorption, radiation and nonappearance of an electric field has been considered.  $B_0$  is exterior magnetic field parameter. Here the flow of the fluid is in the  $\bar{x}$  - way, with  $\bar{z}$  - axis perpendicular to the plate, entire system rotates about the  $\bar{z}$  - axis with a constant vector  $\Omega$ . Supposed that the normal fluid as well as the

suspended nano-particles is in thermal symmetry, in addition no slip takes place between them. Utilizing Boussinesq's approximation, the boundary layer governing equations of the flow are



Physical Model and Co-ordinate system

$$\frac{\partial \bar{w}}{\partial \bar{z}} = 0 \Rightarrow \bar{w} = -w_0 \quad (1)$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + w \frac{\partial \bar{u}}{\partial \bar{z}} - 2\Omega \bar{v} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \frac{[\rho\beta]_{nf}}{\rho_{nf}} g(\bar{T} - \bar{T}_\infty) - \frac{1}{\rho_{nf}} \sigma B_0^2 \bar{u} - \frac{1}{\rho_{nf}} \frac{v_f}{K} \bar{u} \quad (2)$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} + w \frac{\partial \bar{v}}{\partial \bar{z}} + 2\Omega \bar{u} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} - \frac{1}{\rho_{nf}} \sigma B_0^2 \bar{v} - \frac{1}{\rho_{nf}} \frac{v_f}{K} \bar{v} \quad (3)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + w \frac{\partial \bar{T}}{\partial \bar{z}} = \alpha_{nf} \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} - \frac{1}{[\rho C_p]_{nf}} \left( \bar{Q}(\bar{T} - \bar{T}_\infty) + \frac{\partial q_r}{\partial \bar{z}} \right) + \bar{Q}_1 (\bar{C} - \bar{C}_\infty) \quad (4)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + w \frac{\partial \bar{C}}{\partial \bar{z}} = D^* \frac{\partial^2 \bar{C}}{\partial \bar{z}^2} - \bar{K}_1 (\bar{C} - \bar{C}_\infty) \quad (5)$$

The subsequent boundary conditions are

$$\left. \begin{aligned} \bar{u} = 0, \bar{v} = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty \quad & \text{for } \bar{t} \leq 0 \text{ & any } \bar{z} \\ \text{for } \bar{t} \geq 0 \left\{ \begin{aligned} \text{At } y \rightarrow 0: \bar{u} = U_0 \left[ 1 + \frac{\varepsilon}{2} \exp(i \bar{t}) + \exp(-i \bar{t}) \right], \bar{v} = 0, \bar{T} = \bar{T}_w, \bar{C} = \bar{C}_w \\ \text{As } y \rightarrow \infty: \bar{u} \rightarrow 0, \bar{v} \rightarrow 0, \bar{T} \rightarrow \bar{T}_\infty, \bar{C} \rightarrow \bar{C}_\infty \end{aligned} \right. \end{aligned} \right\} \quad (6)$$

The properties of Nano fluids are defined as

$$\left. \begin{aligned} \rho_{nf} - \phi \rho_s &= (1 - \phi) \rho_f, \quad (\rho C_p)_{nf} - (1 - \phi) (\rho C_p)_f = \phi (\rho C_p)_s, \alpha_{nf} (\rho C_p)_{nf} = K_{nf} \\ (\rho\beta)_{nf} - (1 - \phi) (\rho\beta)_f &= \phi (\rho\beta)_s, \mu_{nf} = \frac{\mu_f}{[1 - \phi]^{2.5}}, \frac{K_{nf}}{K_f} = \left[ \frac{K_s + 2K_f - 2\phi(K_f - K_s)}{K_s + 2K_f + 2\phi(K_f - K_s)} \right] \end{aligned} \right\} \quad (7)$$

The thermo physical properties of the base fluid (water) & silver are given below.

Nanofluid $\beta \times 10^5$	$\rho$	$C_p$	$K$
Silver(Ag) 1.89	10,500	235	429
Pure water 21	997.1	4179	0.613

$$q_r = -\frac{4\bar{\sigma}}{3k'_1} \frac{\partial \bar{T}^4}{\partial z} \quad (8)$$

Here

where,  $q_r$  is the radiative flux vector  $\bar{\sigma}$  &  $k'_1$  are respectively the Stefan-Boltzmann constant and the mean absorption coefficient and we suppose that  $\bar{T}^4 = T'_\infty T' - 3T'_\infty^4$  as the difference of temperature within the flow is adequately small. (9)

Introducing dimensionless variables

$$\left. \begin{aligned} u &= \frac{\bar{u}}{U_0}, v = \frac{\bar{v}}{U_0}, z = \frac{\bar{z}U_0}{v_f}, t = \frac{\bar{t}U_0^2}{v_f}, n = \frac{v_f \bar{n}}{U_0^2}, K = \frac{\bar{K} \rho_f U_0^2}{v_f^2}, S = \frac{w_0}{U_0} \\ R &= \frac{2\Omega v_f}{U_0^2}, Q_H = \frac{\bar{Q} v_f^2}{K_f U_0^2}, Q_l = \frac{\bar{Q}_l v_f (C_w - C_\infty)}{(T_w - T_\infty) U_0^2}, F = \frac{4\bar{\sigma} T_\infty^3}{k k'_1}, \text{Pr} = \frac{v_f (\rho C_p)_f}{K_f} \\ \theta &= \frac{\bar{T} - \bar{T}_\infty}{T_w - T_\infty}, \psi = \frac{\bar{C} - \bar{C}_\infty}{C_w - C_\infty}, Sc = \frac{v_f}{D^*}, M = \frac{\sigma B_0^2 v_f}{\rho_f U_0^2}, Kr = \frac{\bar{K} 1}{U_0^2} \end{aligned} \right\} \quad (10)$$

From the Eqs (10) & Eqs. (2) – (5) we obtain:

$$\frac{\partial u}{\partial t} - S \frac{\partial u}{\partial z} - Rv = B_1 \frac{\partial^2 u}{\partial z^2} + B_2 \theta - B_3 u \left[ M + \frac{1}{K} \right] \quad (11)$$

$$\frac{\partial v}{\partial t} - S \frac{\partial v}{\partial z} + Ru = B_1 \frac{\partial^2 v}{\partial z^2} - B_3 v \left[ M + \frac{1}{K} \right] \quad (12)$$

$$\frac{\partial \theta}{\partial t} - S \frac{\partial \theta}{\partial z} = \frac{B_4}{\text{Pr}} \frac{\partial^2 \theta}{\partial z^2} - \frac{B_5 Q_H}{\text{Pr}} \theta + Q_l \psi \quad (13)$$

$$\frac{\partial \psi}{\partial t} - S \frac{\partial \psi}{\partial z} = \frac{1}{Sc} \frac{\partial^2 \psi}{\partial z^2} - Kr \psi \quad (14)$$

The relevant boundary conditions are

$$\left. \begin{aligned} u &= 0, \quad v = 0, \quad \theta = 0, \quad \psi = 0 \quad \text{for} \quad t \leq 0 \quad \& \quad \text{any } z \\ \text{for } t \geq 0 & \left\{ \begin{array}{l} \text{at } y \rightarrow 0: \quad u = \left[ 1 + \frac{\varepsilon}{2} \exp(i \bar{n} t) + \exp(-i \bar{n} t) \right], \quad v = 0, \quad \theta = 1, \quad \psi = 1 \\ \text{as } y \rightarrow \infty: \quad u \rightarrow 0, \quad v \rightarrow 0, \quad \theta \rightarrow 0, \quad \psi \rightarrow 0 \end{array} \right. \end{aligned} \right\} \quad (15)$$

Obviously, the velocity  $U_0$  is elucidate as  $U_0 = \sqrt[3]{g \beta_f v_f (T_w - T_\infty)}$  (16)

$$\frac{\partial \chi}{\partial t} - S \frac{\partial \chi}{\partial z} - Rv = B_1 \frac{\partial^2 \chi}{\partial z^2} + B_2 \theta - B_3 \chi \left[ M + \frac{1}{K} \right] \quad (17)$$

The corresponding boundary conditions are

$$\left. \begin{array}{l}
 \text{at } y \rightarrow 0 : u = \left[ 1 + \frac{\varepsilon}{2} \exp(i \text{int}) + \exp(-i \text{int}) \right], v = 0, \theta = 1, \psi = 1 \\
 \text{as } y \rightarrow \infty : \quad u \rightarrow 0, \quad v \rightarrow 0, \quad \theta \rightarrow 0, \quad \psi \rightarrow 0
 \end{array} \right\} \quad (18)$$

Here  $\chi(z, t) = \bar{u}(z, t) + i\bar{v}(z, t)$  is fluid velocity in the complex form

To solve Eqs. (12)- (15) using Eqs. (17), we suppose that

$$\left. \begin{array}{l}
 \chi(z, t) = \chi_0(z, t) + \frac{\varepsilon}{2} \{ \exp(i n t) \chi_1 + \exp(i n t) \chi_2 \} \dots \\
 \theta(z, t) = \theta_0(z, t) + \frac{\varepsilon}{2} \{ \exp(i n t) \theta_1 + \exp(i n t) \theta_2 \} \dots \\
 \psi(z, t) = \psi_0(z, t) + \frac{\varepsilon}{2} \{ \exp(i n t) \psi_1 + \exp(i n t) \psi_2 \} \dots
 \end{array} \right\} \quad (19)$$

Solving Eqs.(14), Eqs (15) & Eqs.(18) we obtain

$$\chi(z, t) = P_3 e^{-\xi_1 z} + P_4 e^{-\xi_2 z} + P_5 e^{-\xi_3 z} + \frac{\varepsilon}{2} \{ e^{-\xi_4 z} e^{\text{int}} + e^{-\xi_5 z} e^{-\text{int}} \} \quad (20)$$

$$\theta(z, t) = P_1 e^{-\xi_1 z} + P_2 e^{-\xi_2 z} \quad (21)$$

$$\psi(z, t) = e^{-\xi_1 z} \quad (22)$$

The physical quantities of engineering interest are skin-friction co-efficient, local Nusselt number and local Sherwood number are given by:

$$C_f = \left( \frac{\partial \chi}{\partial z} \right)_{z=0} = (-1) \left\{ (P_3 \xi_1 + P_4 \xi_2 + P_5 \xi_3) + \frac{\varepsilon}{2} (\xi_4 \exp(\text{int}) + \xi_5 \exp(-\text{int})) \right\} \quad (23)$$

$$Nu = - \left( \frac{\partial \theta}{\partial z} \right)_{z=0} = P_1 \xi_1 + P_2 \xi_2 \quad (24)$$

$$Sh = - \left( \frac{\partial C}{\partial z} \right)_{z=0} = \xi_1 \quad (25)$$

### III. RESULTS AND DISCUSSION

The effects on the velocity, the temperature as well as the concentration are discussed through graphs.

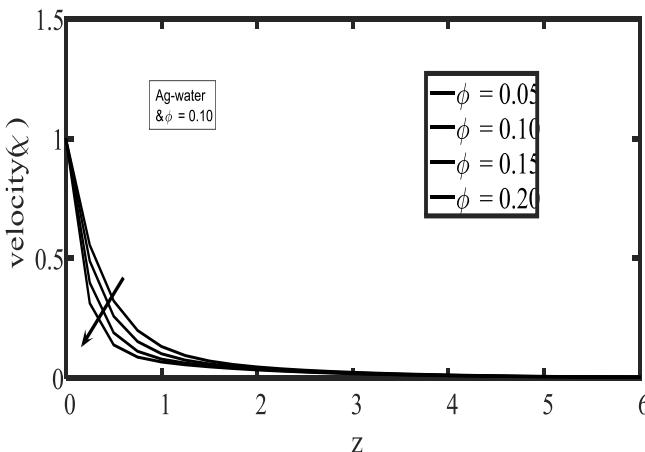


Figure 1: Influence of  $\phi$  on velocity

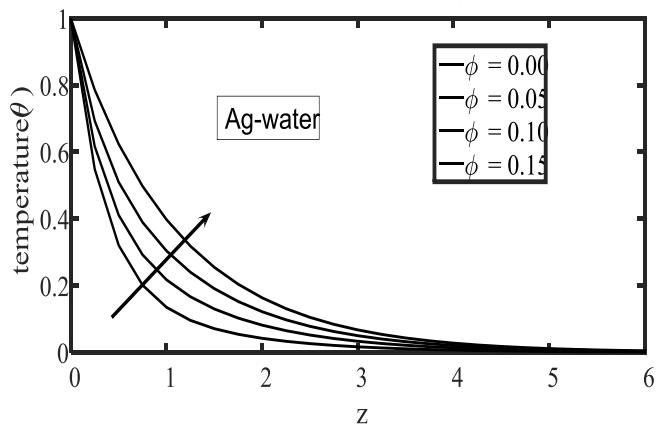
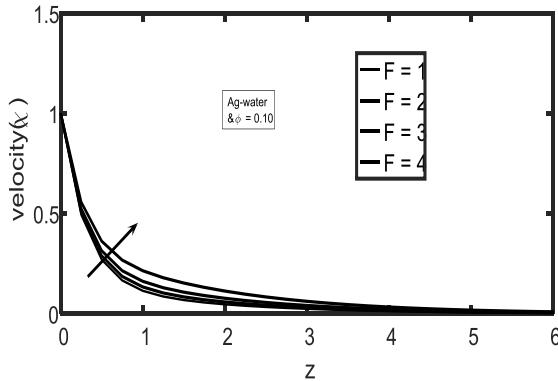
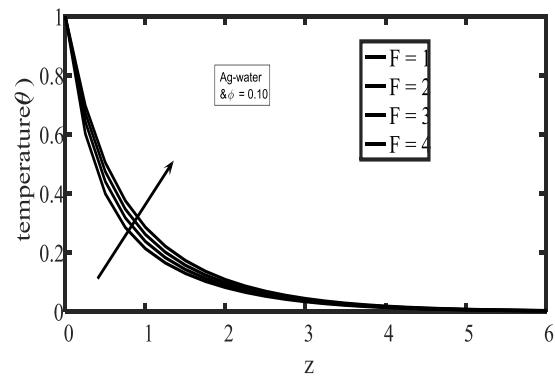


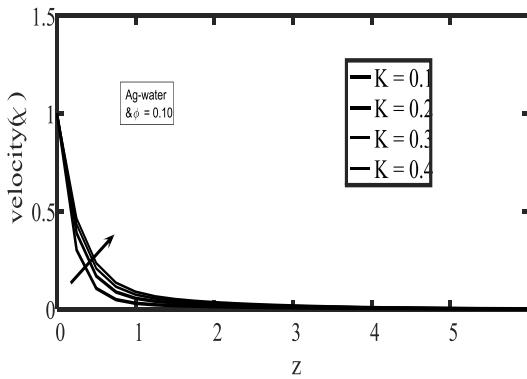
Figure 2: Influence of  $\phi$  on temperature



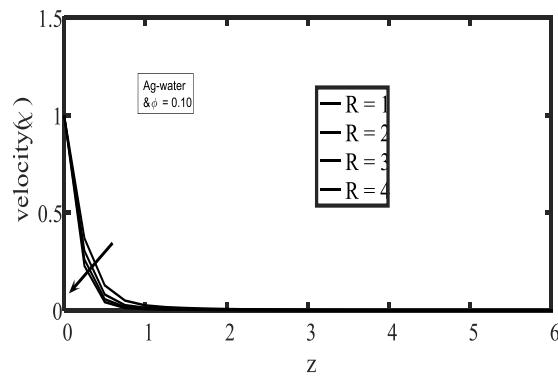
**Figure 3:** Dissimilarity of velocity with  $F$



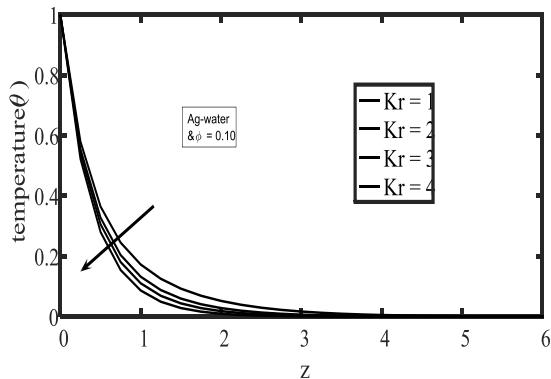
**Figure 4:** Dissimilarity of Temperature with  $F$



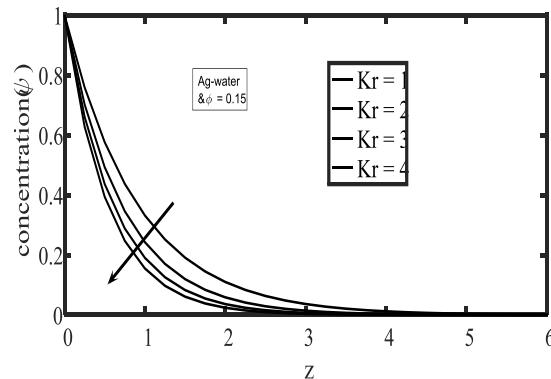
**Figure 5:** Disparity of velocity with  $K$



**Figure 6:** Disparity of velocity with  $R$



**Figure 7:** Disparity of Temperature with  $Kr$



**Figure 8:** Discrepancy of concentration with  $Kr$

Fig 1: & Fig 2: Represents the influence of the Nano particle volume fraction  $\phi$  on the velocity as well as temperature. From this figure it was obvious that different enhancement values of  $\phi$  lead to decline the velocity but reverse effect was occurred in case of temperature. This agrees with the physical behaviour that, when the volume fraction of silver increases, the thermal conductivity increases and then thermal boundary layer increases. Fig 3: & Fig 4: illustrated that the effect of thermal radiation ( $F$ ) in the velocity as well as temperature. From these figures the outcomes indicates that the velocity and temperature rises with the enhancement of thermal radiation ( $F$ ). Owing to the enhancement of  $F$  results to discharge of heat energy as of the flow region and hence temperature rises as the thermal boundary layer thickness turn out to be thin. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid

temperature increases as the thermal boundary layer thickness become thinner. Fig 5: Reflects that the Nano fluid velocity rises with the enhancement of porous parameter ( $K$ ). Rotation parameter  $R$ . Fig 6: Demonstrated that rise of dissimilar values of rotation parameter  $R$  leads to diminished in velocity across the boundary layer. The influence of chemical reaction  $Kr$  on temperature as well concentration is illustrated in the Fig 7 and Fig 8: In this figure the results reflect that concentration as well as temperature declined with the enhancement of  $Kr$ . It is observed that an increase in  $Kr$  contributes to the decrease in the nanofluid temperature distribution.

#### Conflict of Interest

Both the authors have equal contribution in this work and it is declared that there is no conflict of interest for this publication.

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