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Nonlinear Thermal Convective Flow of a Nanofluid over a Convectively Heated Plate with uniform Suction/Injection

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Abstract: *In this paper, the MHD boundary layer flow of a nanofluid along a permeable vertical flat plate with convective boundary condition is studied numerically. We assumed that the relationship between the density and temperature as nonlinear. Buongiorno's nanofluid model, which includes the effects of Brownian motion and thermophoresis, is used in the present study. The system governing of equations are solved using a novel Local Linearization Method (LLM). Validations of the numerical results are verified with the existing literature in some special cases. The influence of various parameters on the flow and physical quantities are presented and discussed.*

Keywords: Nonlinear Thermal Convection, Nanofluid, Suction/Injection Effects, Convective Boundary Condition.

I. INTRODUCTION

Nanofluids are one important non-Newtonian fluids containing nanometer-sized particles (i.e., nanoparticles) of the order of 1nm to 100nm and this theory first used by Choi and Estman (1995). These nanoparticles are made of metals, oxides, carbides, while ethylene glycol, water, mineral oil are used as conventional fluids. Investigations on nanofluids (i.e. a mixture of fluid and nanoparticles) proved that it can be improved thermal conductivity of base fluids. Magneto-hydrodynamics (MHD) has a huge number of industrial applications like as enhanced oil recovery, metal casting, crystal growth and the cooling of nuclear reactors. The extensive literature on the Magneto-hydrodynamics flow of nanofluid over different geometries can be found in the (Chamkha and Aly, 2010; Rana and Bhargava, 2011; Uddin et al., 2012; Satya Narayana et al., 2015) and several therein.

Convective heat transfer analysis has assembled impressive consideration from its significance in environmental technologies and industrial, for example, gas turbines, energy storage, nuclear plants, geothermal reservoirs, rocket propulsion, and so on. As of late, the idea of convective of the convective thermal condition is very famous among the researchers. Since the convective boundary condition has additionally pulled in some intrigue, and this normally is simulated through a Biot number in the wall thermal boundary condition. Aziz and Khan (2012) investigated the boundary layer flow of nanofluid with thermal convective boundary condition. Influence of Biot number and variable suction on water-based nanofluid with a different type of nanoparticles is analyzed by Mutuku-Njane and Makinde (2014).

A large temperature difference between the surface and the ambient fluid compels the researchers to think about the nonlinear density temperature (NDT) variations in the buoyancy force term because of its imperative consequence on fluid flow and heat transfer characteristics. Due to this importance, Vajravelu et al. (2003) studied the nonlinear thermal convection on the flow past a flat porous plate in the presence of blowing/suction. Recently, Kameswaran et al. (2016) investigated the influence of nonlinear thermal convection and thermal stratification on mixed convective flow of nanofluid over a wavy heated surface.

Present work aims to explore the magneto-hydrodynamic flow of nanofluid over a convectively heated permeable plate. The concept of uniform suction/injection effects and nonlinear thermal convection are utilized in mathematical modeling. An aspect of nanoparticles for Brownian motion and thermophoresis is considered. At the zero nanoparticle concentration (Kuznetsov and Nield, 2010) and convective thermal conditions (Aziz and Khan, 2012) are considered. With the help of some appropriate similarity transformations, the nonlinear governing systems are converted into nonlinear ordinary differential systems. The result is gained numerically by Spectral Local Linearization Method (SLLM) (Motsa, 2013; Motsa et al., 2013). The effects of various physical parameters on the non-dimensional velocity, temperature, solid volume fraction, Nusselt number and nanoparticle Sherwood number are presented and discussed.

II. MATHEMATICAL MODELLING

Consider the 2-Dimensional, laminar and steady-state natural convection boundary layer nanofluid flow over a convectively heated vertical plate. The ambient medium temperature and nanoparticle volume fraction are maintained at T_∞ and ϕ_∞ at large values of y. A uniform transverse magnetic field of strength B_0 is applied parallel to the y-axis. The suction/injection velocity distribution is presumed to be v_w. The back side of the vertical plate is in contact with a hot fluid with a temperature T_f which provides a heat transfer coefficient h_f. The nonlinear Boussinesq approximation and standard boundary layer assumptions are employed to simplify the (Buongiorno, 2006) convective transport equations. Then the problem is governed by the following equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{f_\infty}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+\rho_{f_\infty}g\left(1-\phi_\infty\right)\left[\beta_1(T-T_\infty)+\beta_2(T-T_\infty)^2\right]-(\rho_p-\rho_{f\infty})g(\phi-\phi_\infty)-\sigma B_0^{\,2}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial T}{\partial y}\frac{\partial \phi}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=D_B\frac{\partial^2 \phi}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

with corresponding boundary conditions

$$u=0,\ v=v_w,\ -k\frac{\partial T}{\partial y}=h_f(T_f-T),\ D_B\frac{\partial \phi}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial y}=0\ \text{ at }\ y=0$$
$$u=0,\ T=T_\infty,\ \phi=\phi_\infty\ \text{ as }\ y\to\infty \tag{5}$$

Here the dimensional parameters are given by thermophoretic diffusion coefficient as D_T, electric conductivity as σ, thermal diffusivity of the base fluid as α_m, density of the base fluid as ρ_f, Brownian diffusion coefficient as D_B, constant magnetic field of strength as B_0, gravitational acceleration as g, ratio of nanoparticle and base fluid heat capacity as τ. The first and second order thermal expansion coefficients as β_0 and β_1, while density of the particles as ρ_p.

Now, we introduce the stream function, which satisfies the continuity equation automatically and interpreted as $u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$.

Also, introducing dimensionless variables as follows

$$\eta=\frac{y}{x}Ra_x^{1/4},\ \psi=\alpha_m Ra_x^{\,1/4}F(\eta),\ G(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\ S(\eta)=\frac{\phi-\phi_\infty}{\phi_\infty} \tag{6}$$

Reduced Equations:

$$\frac{d^3F}{d\eta^3}+\frac{1}{\Pr}\left[\frac{3}{4}F\frac{d^2F}{d\eta^2}-\frac{1}{2}\left(\frac{dF}{d\eta}\right)^2\right]+(1+\Omega G)G-N_rS-M\frac{dF}{d\eta}=0 \tag{7}$$

$$\frac{d^2G}{d\eta^2}+\frac{3}{4}F\frac{dG}{d\eta}+Nb\frac{dS}{d\eta}\frac{dG}{d\eta}+Nt\left(\frac{dG}{d\eta}\right)^2=0 \tag{8}$$

$$\frac{d^2S}{d\eta^2}+\frac{3}{4}Le\,F\frac{dS}{d\eta}+\frac{Nt}{Nb}\frac{d^2G}{d\eta^2}=0 \tag{9}$$

The associated boundary conditions in terms of new variables become

$$\left.\frac{dF}{d\eta}\right|_{\eta=0}=0,\ F(0)=f_w,\ \left.\frac{dG}{d\eta}\right|_{\eta=0}=-Bi[1-G(0)],\ Nb\left.\frac{dS}{d\eta}\right|_{\eta=0}+Nt\left.\frac{dG}{d\eta}\right|_{\eta=0}=0,$$
$$\left.\frac{dF}{d\eta}\right|_{\eta\to\infty}=0,\ G(\infty)=0,\ S(\infty)=0 \tag{10}$$

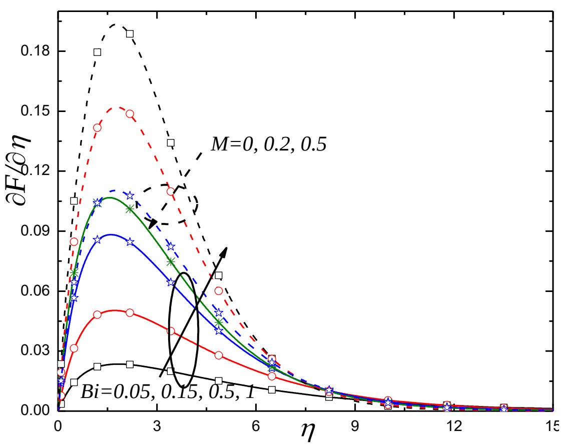
The local Nusselt number ($Nu_xRa_x^{-1/4}$) and local Sherwood number $NSh_xRa_x^{-1/4}$ in non-dimensional form can be written as follows

$$Nu_xRa_x^{-1/4}=-\left.\frac{dG}{d\eta}\right|_{\eta=0}\ \text{and}\ NSh_xRa_x^{-1/4}=-\left.\frac{dS}{d\eta}\right|_{\eta=0} \tag{11}$$

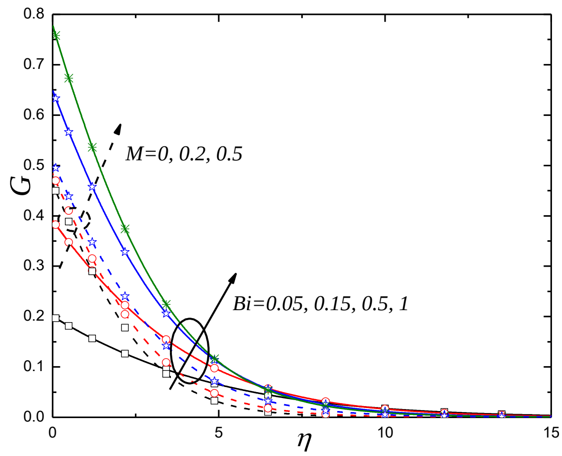
III. RESULTS AND DISCUSSION

Reduced equations (7) to (10) are nonlinear and coupled, and can be solved numerically using Spectral Local Linearization Method (LLM) for different values of parameters such as Biot number (Bi), magnetic parameter (M), suction/injection parameter (f_w), and nonlinear density temperature (Ω). The effects of the emerging parameters on the dimensionless velocity, temperature, nanoparticle concentration, the rate of heat and nanoparticle mass transfer are investigated.

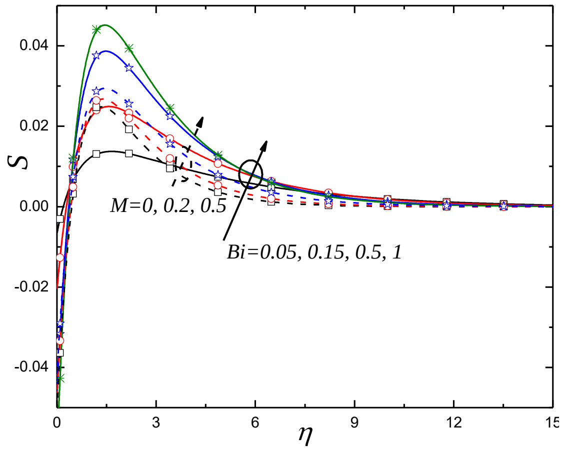
Variations of nanofluid velocity ($dF/d\eta$), temperature (G) and nanoparticle volume fraction (S) for different values of Biot number (Bi) and MHD parameters (M) are depicted in the first of Figs. 1(a)–1(c). It reveals that, an increase in the magnetic parameter leads to decrease the velocity, whereas it increases the thermal and nanoparticle concentration profiles. Moreover, these three fluid profiles are enhanced with a Biot number. Further, the fluid velocity and temperature are more near to the convectively heated plate and the nanoparticle volume fraction will change the direction from negative to positive with respect to both Biot and magnetic parameters.



(a)



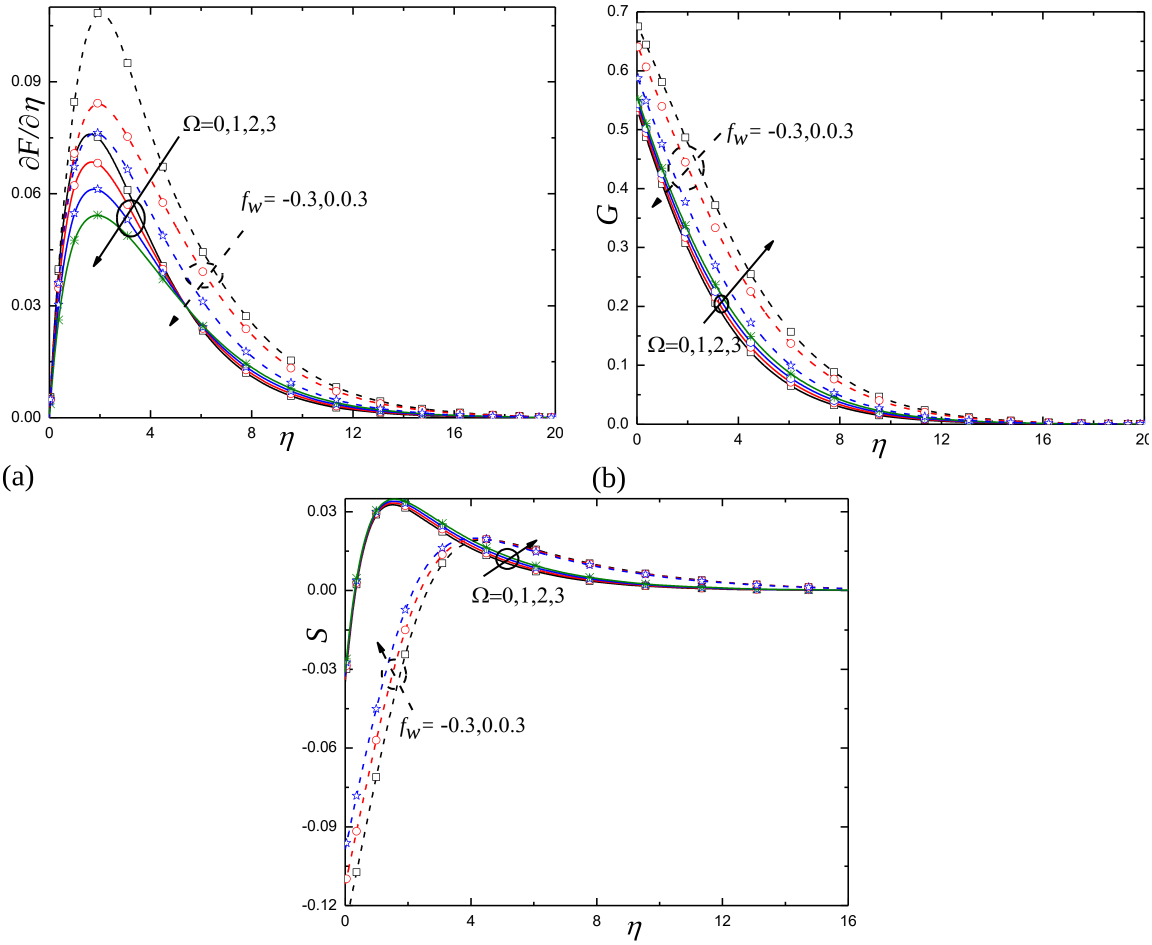
(b)



(c)

Figure 1: Influence of Bi and M on (a) $dF/d\eta$, (b) G, and (c) S

Figure 2(a) to 2(c) emphasizes the changes in above said three profiles with respect to nonlinear density temperature parameter (Ω). From Fig. 2(a) it is observed that, the velocity of nanofluid decreased near the heated surface and increased away from the boundary with the rise of Ω. Also, both temperature and concentration are magnified with Ω. However, both the tangential velocity and temperature profile decreases, and the concentration profile increases with an increase in values of suction/injection parameter (f_w).



(a) (b)

(c)

Figure 2: Influence of Ω and f_w on (a) $dF/d\eta$, (b) G, and (c) S

Table 1: Tabular values of heat and nanoparticle mass transfer rates for Bi, Ω, M and f_w.

		$Nu_x Ra_x^{-1/4}$	$NSh_x Ra_x^{-1/4}$			$Nu_x Ra_x^{-1/4}$	$NSh_x Ra_x^{-1/4}$
Bi	0.05	0.04200027	-0.29106796	M	0	0.07806293	-0.43093592
	0.15	0.10153781	-0.62137677		0.2	0.07717072	-0.44295327
	1.0	0.28294537	-1.50387047		0.5	0.07606908	-0.45793264
Ω	0	0.07455345	-0.47796957	f_w	-0.1	0.06613184	-0.11628278
	1	0.07548095	-0.47027457		0	0.06908445	-0.21913297
	2	0.07617319	-0.46450468		0.1	0.07194092	-0.34625955

Further, the tabular values of heat and nanoparticles mass transfer rates for the above said four parameters are collected in Table (1). From this table, one can notice that both the heat and nanoparticles transfer rates are increased with the nonlinear density temperature parameter, whereas these two transfer rates are decreases with a magnetic parameter. Also, the influence of Biot number and suction/injection parameters on the same physical quantities is shown this table. It reveals that the heat transfer rate of nanofluid is an increasing function of both Biot and suction/injection parameters, whereas the nanoparticle mass transfer rate is a decreasing function of these two parameters.

IV. CONCLUSIONS

Effect of nonlinear thermal convection and uniform suction/injection on the hydromagnetic flow of an incompressible nanofluid along a vertical plate with the convective thermal condition is discussed in the present study. The significant findings of this parametric study are summarized below:

- Magnetic and nonlinear density temperature causes an enhancement of the nanofluid temperature, nanoparticle mass transfer rate, and species concentration, whereas it has the reverse effect on the velocity near the surface
- An increase in suction/injection reduces the velocity and temperature while this has the opposite effect on the concentration and rate of heat transfer at the surface.
- The thickness of velocity, temperature, and nanoparticle volume fraction boundary layer increases with an increase in Biot number.

Conflict of Interest

Both the authors have equal contribution in this work and it is declared that there is no conflict of interest for this publication.

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