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Thermodynamic Analysis for a Cross Diffusive Magnetized Couple Stress Fluid in Horizontal Inner Rotating Cylinder

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Abstract: *The current study analyzes the rate of entropy generation in the MHD flow of couple stress fluid through the horizontal inner rotating cylinder is implemented by using the thermodynamics second law. For the governing equations an analytical result is evaluated using the theory of Modified Bessel's functions. The influence of couple stress parameter, Magnetic parameter, Brinkman number, Soret and Dufour number, Temperature and concentration difference parameters on the velocity, temperature distribution, entropy generation rate and Bejan number are investigated and the results were presented graphically. It is concluded that, the solutal diffusivity has a major conclusion on the rate of entropy generation and it is not an irreversible process of heat in the flow of a couple stress fluid.*

Keywords: MHD flow**,** Thermodynamic analysis**,** Soret and Dufour effects, Couple stress fluid.

I. INTRODUCTION

Throughout the period, the thermodynamics of heat flow exchange have been founded on the analysis of the second law and its structure associated the idea of minimization of entropy generation. The entropy generation and proficiency estimation utilizing the thermodynamics second law are additional dependable than first law-based counts. Bejan (1982),presented the study of minimization of entropy generation in a warm framework, he also clarified that the minimization enhances the effectiveness of a framework by enhancing the exergy. From that point, a few specialists hypothetically considered entropy generation in heat and mass flow frameworks under numerous physical circumstances (Bejan, 1979; Shohel et al., 2003; Ali Mchirgui et al., 2012; Omid et al., 2012; Srinivas et al., 2016; Nagaraju et al., 2017; Srinivas et al., 2017; Sameh et al., 2017; Nagaraju et al., 2018).

Most of the investigations revealed in the writing managed the conventional Newtonian fluids. A few fluids utilized in designing and modern procedures, for example, polymer fluids and melts, suspensions of arrangements and strands, surfactants, slurries, paints, sustenance and restorative items, body fluids, cleansers, inks, natural materials, cements, and so on., display stream properties that can't be clarified by a Newtonian liquid stream demonstrate. To clarify the theological conduct of the above mentioned fluids, various numerical models has been proposed. One among these models is couple stress fluid model presented by Stokes. The couple stress fluid classical has particular highlights, for example, non-symmetric stress tensor and body couples. The couple stress fluids display more confounded volume-subordinate impact than those in Newtonian fluids. A few analysts have contemplated for the most part numerically and scientifically the convective stream of couple stress fluid in the course of rotating cylinders. Kaladhar and

Srinivasacharya (2014) discussed the couple stress moves through the annulus in between two vertical cylinders with chemical reaction. They saw that focus ascends by the chemical reaction parameter upgrade. Dewakar et al. (2017) inspected between concentric cylinders with slip boundary conditions, analytically of some fully developed flows of couple stress fluid. Nagaraju et al.(2015) researched the dissipative, thermocouple stress courses through the two vertical turning cylinders with the lining of porous coating at the external barrel and an uniform outspread attractive field. Adesanya et al. (2017) inspected the thermo dynamics second law rate in the stream of a couple stress fluid with heat and mass transfer. Nagaraju et al. (2017) explored the thermodynamics second law in the stream in the annulur region with porous coating between the two pivoting cylinders beneath the nearness of an outspread attractive field, seeing that speed diminishes with the expanding attractive field parameter. The analysis of the Second law of hydromagnetic couple stress fluid loaded up with non-Darcian porous medium is examined by Opanuga et al. (2017).

After taking inspirations of the examinations and its applications which were used as reference, it is noticed that in order to gauge the energy of magnetized couple stress fluids, the second law of thermo dynamics and irreversibility of mass and heat transfer depends on the entropy generation point is examined. It excludes the impact of mass transfer and cross-diffusion term on rate of thermodynamics second law in the stream couple stress liquid through horizontal cylinders. The subsequent common differential conditions are fathomed utilizing the investigative technique. The correct arrangements are utilized to process the rate of thermodynamics second law and the irreversibility investigation.

The main aim of the current work is to investigate, the Second law analysis of a cross diffusive magnetized couple stress fluid flow through the annular region among horizontal inner rotating cylinder.

II. MATHEMATICAL FORMULATION

Consider an incompressible couple stress fluid flow among horizontal inner rotating infinite cylinders and magnetic field applied Z-direction. Additionally, the effects of Diffusion-thermo (Dufour) and thermal-diffusion(Soret) is considered. The viscous heating effects in the thermal equation are maintained. Here in cylindrical coordinate geometry (r, θ, z) here $\frac{\partial}{\partial\theta}=0$ since the axial symmetry of the flow. The governing flow equations of magnetohydrodynamic, mass and heat transfer of a couple stress fluid (Nagaraju et al., 2018) are as follows

$$\frac{dP}{dR}=\rho\frac{V^2}{R} \quad (1)$$

$$\mu D^2V+\eta_0 D^4V-\sigma B_0^2\,V=0 \quad (2)$$

$$\frac{K_T}{\rho C_p}\left(\frac{d^2T}{dR^2}+\frac{1}{R}\frac{dT}{dR}\right)+\frac{\mu}{\rho C_p}\left(\frac{dV}{dR}-\frac{V}{R}\right)^2+\frac{\eta_0}{\rho C_p}(D^2V)^2+\frac{DK}{C_sC_p}\left(\frac{d^2C}{dR^2}+\frac{1}{R}\frac{dC}{dR}\right)=0 \quad (3)$$

$$D\left(\frac{d^2C}{dR^2}+\frac{1}{R}\frac{dC}{dR}\right)+\frac{DK}{T_m}\left(\frac{d^2T}{dR^2}+\frac{1}{R}\frac{dT}{dR}\right)=0 \quad (4)$$

Where $D^2V=\frac{d^2V}{dR^2}+\frac{1}{R}\frac{dV}{dR}-\frac{V}{R^2}$

The boundary conditions for inner and outer cylinders are taken as

$$(i)\ V=R\Omega,\ D^2V=0, T=T_1, C=C_1\,at\,R=R_1$$
$$(ii\,)V=D^2V=\frac{dT}{dR}=\frac{dC}{dR}=0\quad at\quad R=R_2 \quad (5)$$

The fundamental equations together with the boundary conditions for Eqns. (2)-(4), which currently ends up dimensional less form, are:

$$D^2v+SD^4v-Ha^2(1-\eta)^{-2}\,v=0 \quad (6)$$

$$\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}+Br\left(\frac{dv}{dr}-\frac{v}{r}\right)^2+SBr(D^2v)^2+PrDf\left(\frac{d^2\emptyset}{dr^2}+\frac{1}{r}\frac{d\emptyset}{dr}\right)=0 \quad (7)$$

$$\left(\frac{d^2\phi}{dr^2}+\frac{1}{r}\frac{d\phi}{dr}\right)+Sc\,Sr\left(\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}\right)=0 \quad (8)$$

$$v=\theta=\phi=1\ and\ D^2v=0\text{ at }r=\eta$$
$$\text{(ii)}\quad v=D^2v=\theta'=\phi'=0\quad\text{at}\quad r=1 \quad (9)$$

Where $v=\frac{V}{R_1\Omega}$, $r=\frac{R}{R_2}$, $\eta=\frac{R_1}{R_2}$,
$p=\frac{P}{\rho\Omega^2R_1^2}$, $\theta=\frac{T-T_2}{T_1-T_2}$, $\phi=\frac{C-C_2}{C_1-C_2}$,
$Ha=B_0(R_2-R_1)\sqrt{\frac{\sigma}{\mu}}$, $S=\frac{\eta_0}{\mu R_2^2}$,
$Br=\frac{\mu\Omega^2R_1^2}{K_T(T_1-T_2)}$, $D_f=\frac{DK(C_1-C_2)}{\vartheta C_sC_p(T_1-T_2)}$, $Pr=\frac{\mu C_p}{K_T}$,
$Sc=\frac{\upsilon}{D}$, and $Sr=\frac{DK(T_1-T_2)}{\upsilon T_m(C_1-C_2)}$.

From equation (6), we can get the following equation for *v*:

$$D^4v+\frac{1}{S}D^2v-\frac{Ha^2(1-\eta)^{-2}}{S}v=0 \quad (10)$$

The above equation can be articulated as follows:

$$(D^2 - \lambda_1^2)(D^2 - \lambda_2^2)v = 0 \quad (11)$$

Where $\lambda_1^2 + \lambda_2^2 = \frac{-1}{S}$ And $\lambda_1^2 \lambda_2^2 = -\frac{Ha^2(1-\eta)^{-2}}{S}$.

Since v the velocity is finite in interval η < r < 1, the solution of Equation (10) can be written in the following way:

$$v = a_1 I_1(\lambda_1 r) + a_2 K_1(\lambda_1 r) + a_3 I_1(\lambda_2 r) + a_4 K_1(\lambda_2 r) \quad (12)$$

The constants a_1, a_2, a_3, a_4 can be found by using the no-slip boundary condition and couple stresses vanishing on the boundary(Type A condition) on transverse velocity *v*.

2.1. Second Law Analysis

In the existence of Joule Heating, the volumetric entropy generation number can be expressed as

$$S_G = \frac{K_T}{T_1^2}(\nabla T)^2 + \frac{\mu}{T_1}\varphi + \frac{J^2}{\sigma T_1} + \frac{RD}{C_1}(\nabla C)^2 \quad (13)$$

In Equation(13), φ is the viscous heating, J is the current density ΔT is temperature difference, ΔC is Concentration difference, R is ideal gas constant.

$$S_G = \frac{K_T}{T_1^2}\left(\frac{\Delta T}{R_2}\right)^2\left(\frac{d\theta}{dr}\right)^2 + \frac{\mu R_1^2 \Omega^2}{R_2^2 T_1}\left(\frac{dv}{dr} - \frac{v}{r}\right)^2 + \frac{\eta_0 R_1^2 \Omega^2}{R_2^4}(D^2 v)^2 + \frac{\sigma R_1^2 \Omega^2 B_0^2}{T_1} v^2 + \frac{RD}{C_1}\left(\frac{\Delta c}{R_2}\right)^2\left(\frac{d\emptyset}{dr}\right)^2 \quad (14)$$

The non-dimensional form of the Entropy generation Ns of Eqn. (14) can be given as

$$Ns = \frac{R_2^2 T_1^2}{K_T (\Delta T)^2} S_G$$

$$Ns = \left(\frac{d\theta}{dr}\right)^2 + \frac{Br}{T_d}\left[\left(\frac{dv}{dr} - \frac{v}{r}\right)^2 + S(D^2 v)^2\right] + Ha^2 (1-\eta)^{-2} \frac{Br}{T_d} v^2 + \lambda \left(\frac{C_d}{T_d}\right)^2 \left(\frac{d\emptyset}{dr}\right)^2$$

$$Ns = N_H + N_F + N_M + N_D \quad (15)$$

Where $T_d = \frac{\Delta T}{T_1}$, $C_d = \frac{\Delta C}{C_1}$ and $\lambda = \frac{RDC_1}{K_T}$.

2. 2. Bejan Number

Bejan number is described to be a fraction of entropy generation due to the heat transfer.

$$Be = \frac{1}{1 + \frac{N_F + N_M + N_D}{N_H}} = \frac{1}{1+\emptyset} \quad (16)$$

III. RESULTS AND DISCUSSION

A logical model for a cross diffusive Magnetized couple stress fluid in the horizontal inner rotating cylinder has been produced. To know the behaviour of the fluid qualities, Velocity(v), Temperature (θ), Concentration(ϕ), Entropy age number(*Ns*), Bejan number(*Be*), for various parameters like Couple stress parameter (*S*), Hartmann number (*Ha*), Soret parameter (*Sr*), Dufour parameter (D_f), Concentration difference number(C_d), Brinkmann number(*Br*), Temperature distinction number(T_d) are determined and are portrayed through diagrams.

Fig. 1(a) - 1(b) exhibit the impact of azimuthal velocity (*v*) concerning Hartmann number (*Ha*) and couple pressure parameter (*S*). It is seen that the velocity increments with *Ha* and diminishes with *S*. Fig. 2(a) depict the impact of temperature by changing the Dufour number. It clears that, there is a steady decline in the temperature with increment in Dufour number. Fig. 2(b) demonstrates the variety of concentration on Soret number (*Sr*). It is seen that there is an expansion in concentration with increment in *Sr*. Fig 3(a) - 3(b) gives the impact of temperature difference number(T_d) on the Bejan number and the entropy generation number. It is noted that the entropy generation number increments close to the internal cylinder as the temperature difference number builds; Bejan number abatements close to the axis of the cylinder (i.e., *Be* increment as *r* increment up to *r* = 0.2) as T_d increment after that Be stays constant. Fig. 4(a) - 4(b) demonstrates the response of C_d number on the Bejan number and entropy generation number. The impact of C_d is irrelevant on *Ns* (fig.4a), additionally it is seen that C_d expands Bejan number. Fig. 5(a) - 5(c) demonstrate the impact of Brinkman numbers on temperature, Bejan number and entropy generation number. It is noted that as entropy generation number and Bejan number increment as Brinkmann number builds, and temperature diminishes concerning *r*.

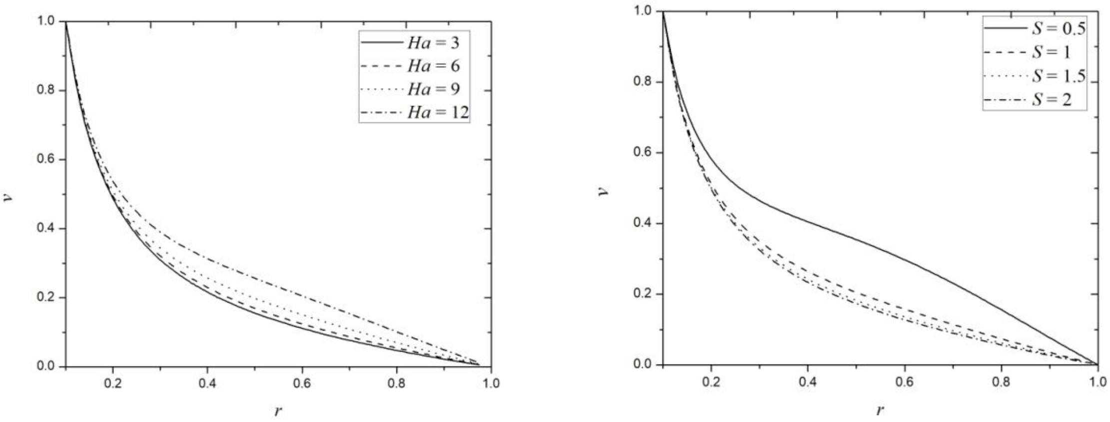

Figure 1: (a)-1(b) Variation of Hartmann number (*Ha*) and couple stress parameters (*S*) on velocity (*v*) with respect *r*

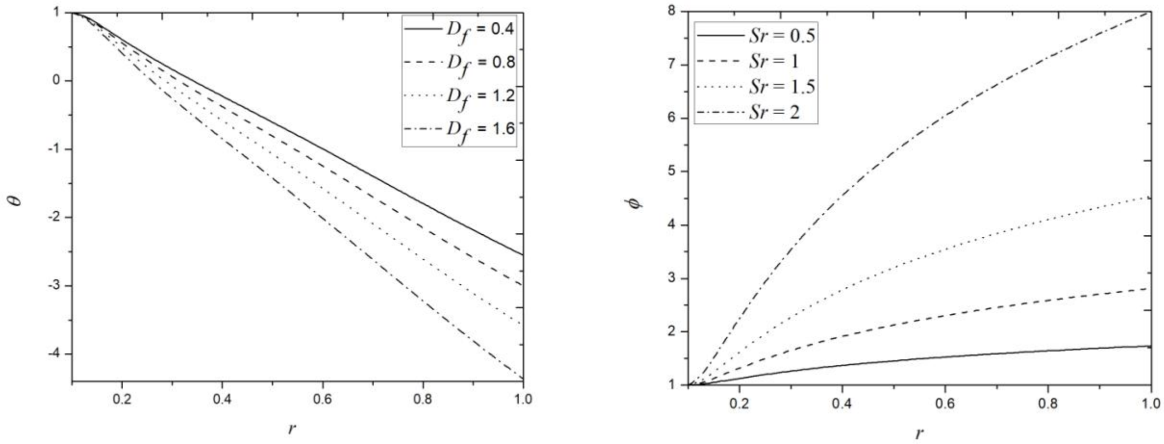

Figure 2: (a)-2(b) variation of Dufour parameter on Temperature and effect of Soret parameter on concentration.

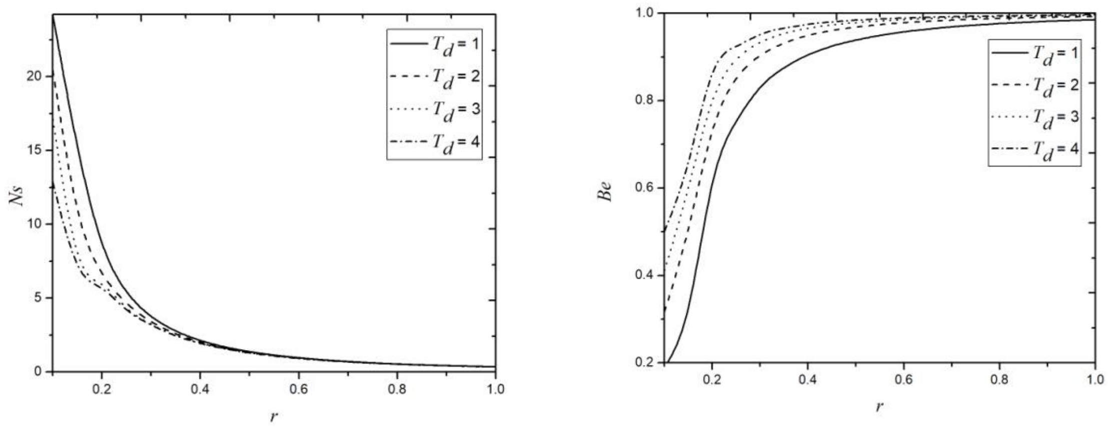

Figure 3: (a)-3(b) Variation of temperature difference number on entropy generation number and Bejan number.

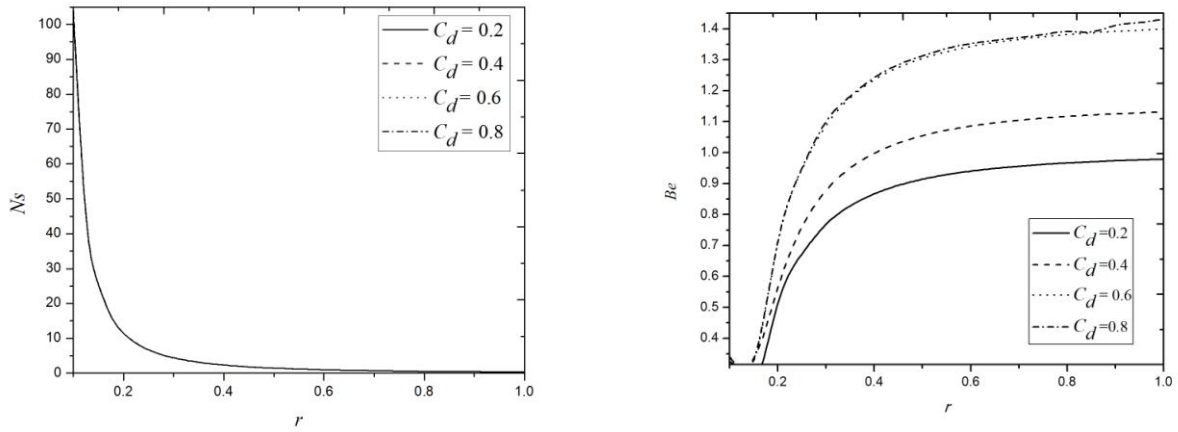

Figure 4: (a)-4(b) Variation of concentration difference number on entropy generation number and Bejan Number

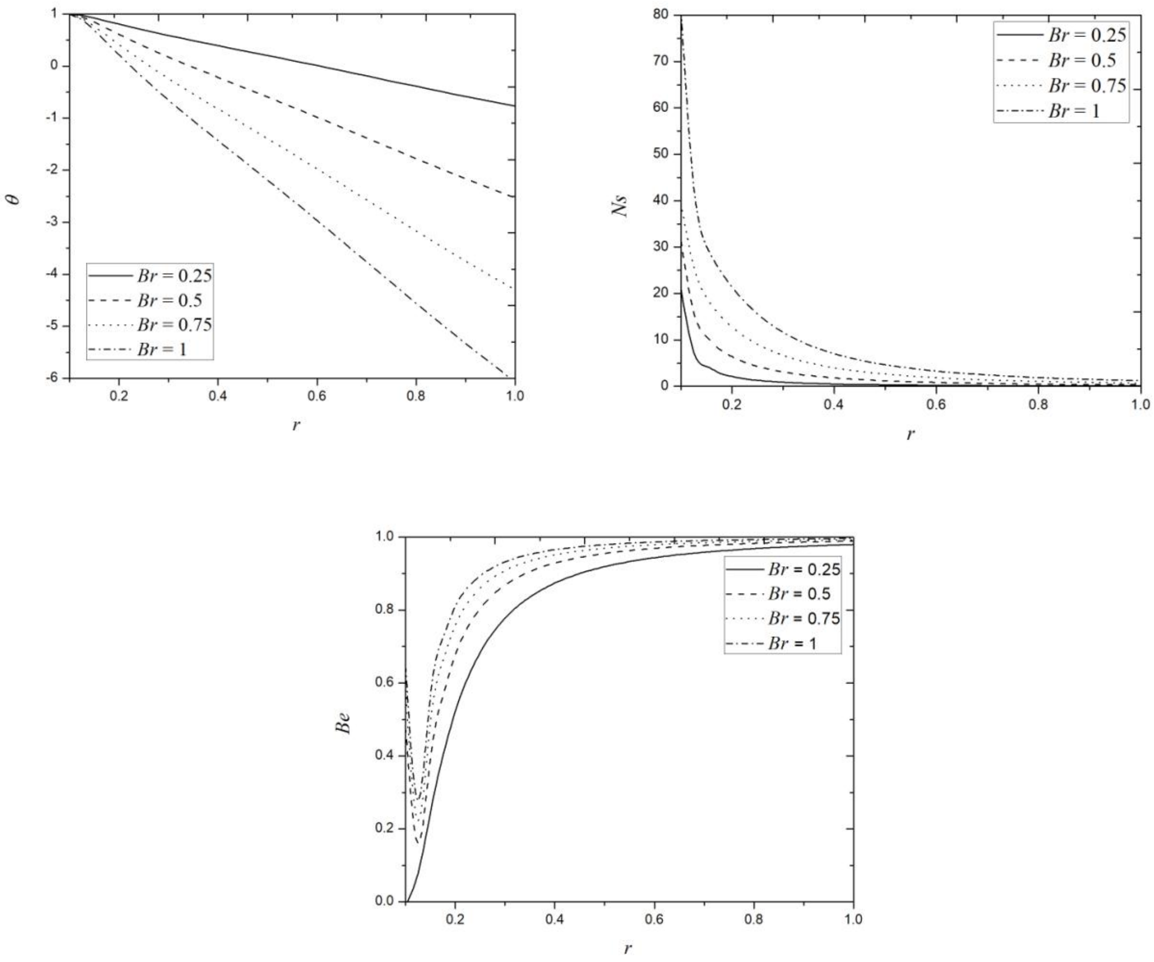

Figure 5: (a)-5(c) Variation of Brinkman number on temperature, Entropy generation number and Bejan number.

IV. CONCLUSION

In the present research work, the investigative arrangement acquired by utilizing adjusted Bessel's elements of a magnetized couple stress fluid flow through the horizontal inner rotating cylinders has been talked about.

1. There is a consistent decline in the temperature as Dufour and Brinkman number increment.
2. Entropy number has a decrement and stays consistent with the rise of the Temperature difference parameter, no critical change as there is an addition in Concentration difference number.
3. Bejan number reductions with the expansion of Concentration difference number, Brinkman number.

Conflict of Interest

Both the authors have equal contribution in this work and it is declared that there is no conflict of interest for this publication.

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