International Journal of Advanced Trends in Computer Applications
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PERISTALTIC PUMPING OF 4th GRADE FLUID THROUGH POROUS MEDIUM IN AN INCLINED CHANNEL WITH VARIABLE VISCOSITY UNDER THE INFLUENCE OF SLIP EFFECTS

[1]Mohd.Riyazuddin, [2]Y.V.K. Ravi Kumar, [3]M.V.Ramana Murthy
[1]Department of Mathematics,
Keshav Memorial Institute of Technology, Hyderabad – 500 029, INDIA
[2]Practice School Division,
Birla Institute of Technology and Science,– Pilani, Rajasthan, INDIA.
[3]Department of Mathematics,
Mahatma Gandhi Institute of Technology, Gandipet, Hyderabad – 500 075, INDIA.
[1]riyazuddin17@gmail.com, [2]yvk.ravikumar@pilani.bits-pilani.ac.in, [3]mv.rm50@gmail.com

Abstract: *This article deals with the peristaltic pumping of 4^{th} grade fluid through porous medium in an inclined channel with variable viscosity under the influence of slip condition. In the acquired solution expressions the long wavelength & low Reynolds number assumptions are utilized and it is noticed that we observed that pressure rise increases with the increase in* $\gamma, \Gamma\ \sigma, \beta$ *and axial velocity decreases with increase in values of* Γ, β, ϕ .

Keywords: 4^{th} grade fluid, porous channel, slip condition, peristaltic pumping.

I. INTRODUCTION

Shehawes [1] considered “influence on porous boundaries on peristatic pumping through a porous passage with pulsatile Magneto fluid with a porous channel" corresponding results are obtained by Gad N.S. et al. [2]. Chaube et al. [3] considered “slip effect on peristatic pumping of micropolor fluid”. Misery et al. [4] considered “effect of a fluid with variable viscosity”. Hussain [5] studied “slip effect on the peristatic pumping of MHD fluid with variable viscosity”. Ravi Kumar [6] studied “pulsatile pumping of a viscous strafed fluid of variable viscosity”. Hayat T et al. [7] studied “Soret and Dufour effect on flow MHD fluid with variable viscosity”. Provost A [8] studied “flow of a second-order fluid in a planar passage” & corresponding results in tubes are obtained by Schwarz W.H [9]. Masuoka Stokes et al. [10] investigated “1^{st} problem for a second grade fluid in a porous half space”. Ali N [11] studied “peristaltically induced motion of a MHD third grade fluid”. Harojun [12] studied “effect of peristatic transport of a 3rd order fluid in an asymmetric passage”. Qureshi [13] investigated the “influence of slip on the peristaltic pumping of a 3rd order fluid”. Sreenath [14] studied “peristaltic flow of a 4th grade fluid between porous walls”.

We intend to study peristaltic flow of 4^{th} grade fluid across porous medium in an inclined channel with variable viscosity under the influence of slip situation.

II. MATHEMATICAL MODEL

Equation of flow of 4th grade fluid passing through porous medium

$$\rho \frac{d\bar{V}}{dt} = -grad\ p + div\,\bar{S} - \frac{\mu}{\bar{k}}\bar{V} \tag{1}$$

where

ρ : Density

$\bar{V}$: Velocity vector

$\bar{k}$: Permeability of the porous medium

p : Pressure and

$\frac{d}{dt}$: Material derivative

The law of conservation of mass is defined by

$$div\bar{V} = 0 \tag{2}$$

4th grade fluid stress tensor $\bar{S}$ is

$$\bar{S} = \mu \bar{A}_1 + \alpha_1 \bar{A}_2 + \alpha_2 \bar{A}_1^2 + \beta_1 \bar{A}_3 + \beta_2\left(\bar{A}_2 \bar{A}_1 + \bar{A}_1 \bar{A}_2\right) + \beta_3\left(tr\,\bar{A}_1^2\right)\bar{A}_1 + \gamma_1 \bar{A}_4 + \gamma_2(\bar{A}_3\bar{A}_1 + \bar{A}_1\bar{A}_3) + \gamma_3\bar{A}_2^2 + \gamma_4(\bar{A}_2\bar{A}_1^2 + \bar{A}_1^2\bar{A}_2) + \gamma_5(tr\bar{A}_2)\bar{A}_2 + \bar{A}_2 + \gamma_6(tr\bar{A}_2)\bar{A}_1^2 + [\gamma_7(tr\bar{A}_3 + \gamma_8 tr(\bar{A}_2\bar{A}_1))]\bar{A}_1 \tag{3}$$

where $\mu, \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7, \gamma_8$ being material constants and A_n is the Rivlin – Ericksen tensors defined

$$\bar{A}_{n+1} = \frac{d\bar{A}_n}{dt} + \bar{A}_n(grad\bar{V}) + (grad\bar{V})^T \bar{A}_n, \quad n > 1 \tag{4}$$

$$\bar{A}_1 = (grad\bar{V}) + (grad\bar{V})^T \tag{5}$$

Flow of 4th grade fluid is studied in a uniform channel with porous medium of width 2d, inclined at an angle γ with variable viscosity

$$H(\bar{X},\bar{t}) = d + b\,\mathrm{Cos}\left(\frac{2\pi}{\lambda}\left(\bar{X} - c\bar{t}\right)\right) \tag{6}$$

where

b : Wave amplitude

λ : Wave distance

$\bar{t}$: Time

$\left(\bar{U},\bar{V}\right)$, $\left(\bar{u},\bar{v}\right)$: Velocity components in fixed & wave frame respectively and the relation between fixed and wave frames as

$$\bar{x} = \bar{X} - c\,\bar{t},\ \bar{y} = \bar{Y}, \quad \bar{u} = \bar{U} - \bar{c},\ \bar{v} = \bar{V} \tag{7}$$

when we non dimensionalize eqns (3), (6) &(7)

$$S = A_1 + \lambda_1 A_2 + \lambda_2 A^2{}_1 + \xi_1 A_3 + \xi_2\left(A_2 A_1 + A_1 A_2\right) + \xi_3\left(tr\,A^2{}_1\right)A_1 + \eta_1 A_4 + \eta_2(A_3A_1 + A_1A_3) + \eta_3 A_2^2 + \eta_4(A_2A_1^2 + A_1^2A_2) + \eta_5(trA_2)A_2 + A_2 + \eta_6(trA_2)A_1^2 + [\eta_7(trA_3 + \eta_8 tr(A_2A_1))]A_1 \tag{8}$$

$$h(x) = 1 + \phi\cos 2\pi x \tag{9}$$

$$y = \frac{\bar{y}}{d}, \quad u = \frac{\bar{u}}{c}, \quad v = \frac{\bar{v}}{\delta\,c}, \quad h = \frac{\bar{H}}{d},$$

$$S = \frac{d}{\mu_0\,c}\bar{S}\left(\bar{x}\right), \quad \mu(y) = \frac{\mu(\bar{y})}{\mu_0}, \quad \delta = \frac{d}{\lambda},$$

$$\mathrm{Re} = \frac{\rho\,c\,d}{\mu_0}, \quad Fr = \frac{c^2}{g\,d}, \quad \sigma^2 = \frac{d^2}{k}, \quad \lambda_1 = \frac{\alpha_1\,c}{\mu_0\,d},$$

$$\lambda_2 = \frac{\alpha_2\,c}{\mu_0\,d}, \quad \xi_1 = \frac{\beta_1\,c^2}{\mu_0\,d^2}, \quad \xi_2 = \frac{\beta_2\,c^2}{\mu_0\,d^2},$$

$$\xi_3 = \frac{\beta_3\,c^2}{\mu_0\,d^2}, \quad \phi = \frac{b}{d}, \quad \eta_1 = \frac{\gamma_1 c^3}{\mu a^3}, \quad \eta_2 = \frac{\gamma_2 c^3}{\mu a^3},$$

$$\eta_3 = \frac{\gamma_3 c^3}{\mu a^3}, \quad \eta_4 = \frac{\gamma_4 c^3}{\mu a^3}, \quad \eta_5 = \frac{\gamma_5 c^3}{\mu a^3},$$

$$\eta_6 = \frac{\gamma_6 c^3}{\mu a^3}, \quad \eta_7 = \frac{\gamma_7 c^3}{\mu a^3}, \quad \eta_8 = \frac{\gamma_8 c^3}{\mu a^3}. \tag{10}$$

where

δ : Wave no.,

Re: Reynolds no.,

ϕ : Amplitude ratio,

σ^2 : Porosity parameter,

$\lambda_i\left(i = 1,2\right)$: Non-Newtonian parameters and

$\xi_i\left(i = 1,2,3\right)$: Non-Newtonian parameters.

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \delta\frac{\partial S_{xx}}{\partial y} + \frac{\partial S_{xy}}{\partial y} - \mu(y)\,\sigma^2(u+1) + \frac{\mathrm{Re}}{Fr}\mathrm{Sin}\,\gamma \tag{11}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial S_{xy}}{\partial x} + \delta\frac{\partial S_{yy}}{\partial y} + \delta^2\,\mu(y)\,\sigma^2\,v - \delta\frac{\mathrm{Re}}{Fr}\mathrm{Cos}\,\gamma \tag{12}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \left(\frac{\partial^2}{\partial y^2} - \delta^2\frac{\partial^2}{\partial y^2}\right)S_{xy} + \delta\frac{\partial^2}{\partial x\partial y}\left(S_{xx} + S_{yy}\right) - \sigma^2\left(\frac{\partial}{\partial y}\left(\mu(y)(u+1)\right) + \delta^2\frac{\partial v}{\partial x}\right) \tag{13}$$

applying the long wave distance & less Re. number approximations we obtain

$$\frac{\partial p}{\partial x} = \frac{\partial S_{xy}}{\partial y} - \mu(y)\,\sigma^2(u+1) + \frac{\mathrm{Re}}{Fr}\mathrm{Sin}\,\gamma \tag{14}$$

$$\frac{\partial p}{\partial y} = 0 \tag{15}$$

$$S_{xy} = \mu(y)\frac{\partial u}{\partial y} + 2\Gamma\left(\frac{\partial u}{\partial y}\right)^3 \tag{16}$$

where $\Gamma = \xi_2 + \xi_3$ is the Deborah number.

Eq. (15) indicates that $p \neq p(y)$.

From equation (13) we have

$$\frac{\partial^2}{\partial y^2}\left(\mu(y)\frac{\partial u}{\partial x}+2\Gamma\left(\frac{\partial u}{\partial y}\right)^3\right)$$

$$-\sigma^2\left(\frac{\partial}{\partial y}\left(\mu(y)(u+1)\right)+\delta^2\frac{\partial v}{\partial x}\right)=0 \quad (17)$$

The boundary conditions are,

$$\frac{\partial u}{\partial y}=0 \quad \text{at} \quad y=0 \quad (18)$$

$$u+\beta\left(\mu(y)\frac{\partial u}{\partial y}+2\Gamma\left(\frac{\partial u}{\partial y}\right)^3\right)=-1 \quad \text{at} \quad y=h \quad (19)$$

where $\beta=\frac{l}{d}$ is a slip parameter

the effect of variable viscosity on peristaltic flow

$$\mu(y)=e^{-\alpha y}, \quad \alpha<<1 \quad (20)$$

i.e. $\mu(y)=1-\alpha\, y+\mathrm{O}(\alpha^2)$ (21)

The volume flow rate q is

$$q=\int_0^h u\,dy \quad (22)$$

The instantaneous flow Q (X , t) is

$$Q(X,t)=\int_0^h U\,dY=\int_0^h (u+1)dy=q+h \quad (23)$$

The time averaged volume flow rate $\bar{Q}$ over is

$$\bar{Q}=\frac{1}{T}\int_0^T Q\,dt=q+1 \quad (24)$$

The pressure rise Δp

$$\Delta p=\int_0^1\frac{dp}{dx}dx \quad (25)$$

III. SOLUTION OF THE PROBLEM PERTURBATION SOLUTION

We use the below equations in order to obtain solution

$$u=u_0+\Gamma u_1+.....$$

$$p=p_0+\Gamma\, p_1+..... \quad (26)$$

where

$$u_0=u_{00}+\alpha\, u_{01}+.....$$

$$u_1=u_{10}+\alpha\, u_{11}+.....$$

$$p_0=p_{00}+\alpha\, p_{01}+..... \quad (27)$$

$$p_1=p_{10}+\alpha\, p_{11}+.....$$

when we put Eq. (27) into Eqs. (17) to (25) and separate the terms of differential order in Γ and α we obtain

3.1 Zero-order Equation:

$$\frac{\partial p_{00}}{\partial x}=\frac{\partial^2 u_{00}}{\partial y^2}-\sigma^2(u_{00}+1)+\frac{\mathrm{Re}}{Fr}\mathrm{Sin}\,\gamma \quad (28)$$

$$\frac{\partial u_{00}}{\partial y}=0 \quad \text{at} \quad y=0 \quad (29)$$

$$u_{00}+\beta\frac{\partial u_{00}}{\partial y}=-1 \quad \text{at} \quad y=h \quad (30)$$

3.2 First-order Equation:

$$\frac{\partial p_{01}}{\partial x}=\frac{\partial^2 u_{01}}{\partial y^2}-\sigma^2 u_{01}+\sigma^2\, y(u_{00}+1)-\frac{\partial}{\partial y}\left(y\frac{\partial u_{00}}{\partial y}\right) \quad (31)$$

$$\frac{\partial u_{01}}{\partial y}=0 \quad \text{at} \quad y=0 \quad (32)$$

$$u_{01}-\beta y\frac{\partial u_{00}}{\partial y}+\beta\frac{\partial u_{01}}{\partial y}=0 \quad \text{at} \quad y=h \quad (33)$$

3.3 Zero-order Solution

Solving equation (28) we get,

$$u_{00}=c_1\,Cosh(\sigma h)-\frac{G_0}{\sigma^2} \quad (34)$$

where

$$c_1=\frac{1}{L_1}\left(\frac{G_0}{\sigma^2}-1\right)$$

$$L_1=Cosh(\sigma h)+\beta\,\sigma\,Sinh(\sigma h)$$

$$G_0=\frac{\partial p_0}{\partial x}+\sigma^2-\frac{Re}{Fr}Sin(\gamma)$$

3.4 First-order Solution

Sub. zero order soln. Eq. (34) in Eq. (31) and then solving the
Eq. (31), we get

$$u_{01}=c_3\,Cosh(\sigma h)+c_4\,Sinh(\sigma h)$$

$$-\frac{(G_1+y)}{\sigma^2}+\frac{c_1\,y}{2}Cosh(\sigma h) \quad (35)$$

$$G_1=\frac{\partial p_{01}}{\partial x}$$

$$L_2=Sinh(\sigma h)+\beta\,\sigma\,Cosh(\sigma h)$$

$$L_3=\frac{c_1}{2}\left((h+\beta)Cosh(\sigma h)-\beta\,\sigma\,h\,Sinh(\sigma h)\right)$$

$$L_4=\frac{G_1+h+\beta}{\sigma^2}$$

$$c_3=\frac{1}{L_1}\left(L_4-\frac{c_1L_3}{2}-c_4\,L_2\right)$$

$$c_4 = \frac{1}{\sigma^3} - \frac{c_1}{2\sigma}$$

applying equations (34) & (35) with the relation $u = u_{00} + \Gamma\, u_{01}$ we get

$$u = \frac{1}{\sigma^2}\left(\frac{\partial p}{\partial x} + \sigma^2 + \frac{Re}{Fr} Sin(\gamma)\right)\left(\frac{Cosh(\sigma h)}{L_1} - 1\right) - \frac{Cosh(\sigma h)}{L_1}$$
$$+\Gamma\left(\begin{array}{l} \frac{Cosh(\sigma h)}{L_1}\left(\frac{h+\beta}{\sigma^2} + \frac{L_3}{2} - \frac{L_2}{2\sigma^3} - \frac{G_0}{2\sigma^2}\left(L_3 + \frac{L_2}{\sigma}\right)\right) \\ + \frac{1}{2L_1}\left(\frac{G_0}{\sigma^2} - 1\right)\left(\frac{Sinh(\sigma h)}{\sigma} + y\, Cosh(\sigma h)\right) \\ - \frac{Sinh(\sigma h)}{\sigma^3} - \frac{y}{\sigma^2} \end{array}\right) \qquad (36)$$

The volume flow rate is

$$q = \int_0^h u \; dy$$

$$q = \frac{1}{\sigma^2}\left(\frac{\partial p}{\partial x} + \sigma^2 + \frac{Re}{Fr} Sin(\gamma)\right)\left(\frac{Sinh(\sigma h)}{\sigma L_1} - h\right) - \frac{Sinh(\sigma h)}{\sigma L_1}$$
$$+\Gamma\left(\begin{array}{l} \frac{Sinh(\sigma h)}{\sigma L_1}\left(\frac{h+\beta}{\sigma^2} + \frac{L_3}{2} - \frac{L_2}{2\sigma^3} - \frac{G_0}{2\sigma^2}\left(L_3 + \frac{L_2}{\sigma}\right)\right) \\ + \left(\frac{G_0}{\sigma^2} - 1\right)\frac{h\, Sinh(\sigma h)}{2\sigma L_1} - \frac{Cosh(\sigma h)}{\sigma^4} - \frac{h^2}{2\sigma^2} + \frac{1}{\sigma^4} \end{array}\right) \qquad (37)$$

The pressure rise is

$$\Delta p = \int_0^1 \frac{dp}{dx}\, dx$$

where

$$\frac{dp}{dx} = \frac{\sigma^3 L_1}{Sinh(\sigma h) - h\sigma L_1}\left(q + \frac{Sinh(\sigma h)}{\sigma L_1} - \Gamma\left(\begin{array}{l} \frac{Sinh(\sigma h)}{\sigma L_1}\left(\begin{array}{l}\frac{h+\beta}{\sigma^2} + \frac{L_3}{2} - \frac{L_2}{2\sigma^3} \\ - \frac{G_0}{2\sigma^2}\left(L_3 + \frac{L_2}{\sigma}\right)\end{array}\right) \\ + \frac{h\, Sinh(\sigma h)}{2\sigma L_1}\left(\frac{G_0}{\sigma^2} - 1\right) \\ - \frac{Cosh(\sigma h)}{\sigma^4} \\ - \frac{h^2}{2\sigma^2} + \frac{1}{\sigma^4} \end{array}\right)\right)$$
$$-\sigma^2 - \frac{Re}{Fr} Sin(\gamma) \qquad (38)$$

IV. RESULTS AND DISCUSSION

Fig. 1 to 5 show that the changes in pressure rise due to different parameters.
In Fig. 1 to Fig. 5, it is noticed that pressure rise increases with the increase in γ, Γ. In Fig.2 and Fig.4 we observe that the pressure rise increases with decrease in values of ϕ, σ. Fig.3 shows that the pressure rise increases with decrease in values of $\beta\ (\Delta p < 0)$ and it is observed that pressure rise increases with decrease in values of $\beta\ (\Delta p > 0)$.
Fig. 6 to Fig. 10 show the changes on pressure gradient due to different parameters.
Fig. 7 to 8 show graphs for pressure gradient vs x. It can be noticed that there is a rise in pressure gradient with a rise in σ, β in first half wave length of the passage and the effect reverses in second half wave length of the passage. Fig. 6, Fig. 9 and Fig. 10 show graphs for pressure gradient vs x. We can notice that there is a rise in pressure gradient with lowering in values of Γ, ϕ, γ in first half wave length of the passage and the effect reverses in second half wave length of the passage. Fig. 11 to 15 show the effects of axial velocity due to various parameters.

Fig. 11, 13 and 14 show velocity vs y. It is noticed that there is a fall in velocity with a rise in values of Γ, β, ϕ for a stable y value. In Fig. 15 it is noticed that velocity rises with rise in γ for stable y value. In Fig. 12 it is noticed that there is a fall in velocity with rise in the porosity parameter σ.

4.1 Trapping:

Fig. 16 show the stream line patterns and trapping for various values of the β and here we noticed that the size of the bolus decreases with rise in β. Fig. 17 show the stream line patterns and trapping for various values of Γ, here we observed that the size of bolus falls with rise in Γ. Fig. 18 show the stream line patterns and trapping for various values of γ, here we observed that the size of bolus increases with rise in γ. Fig. 19 show the stream line patterns and trapping for various values of σ, here we noticed that the size of the bolus increases with increase of σ.

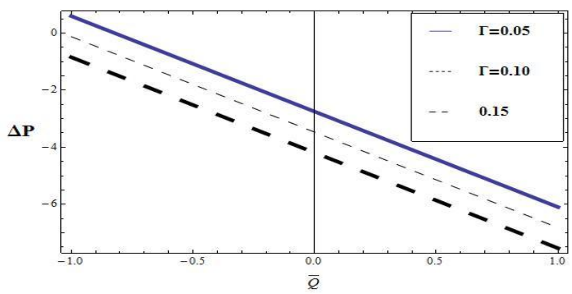


Fig. 1 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'Γ'.

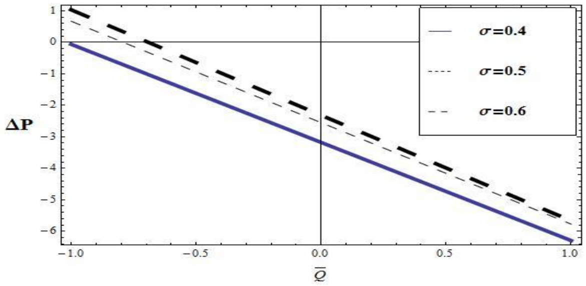


Fig. 2 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'σ'.

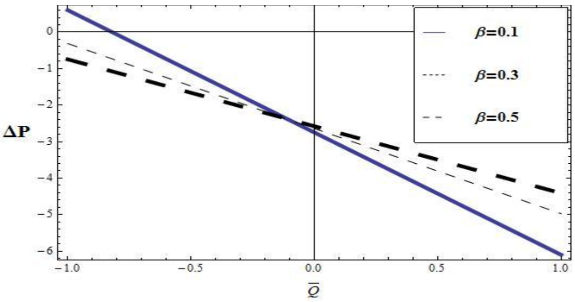


Fig. 3 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'β'.

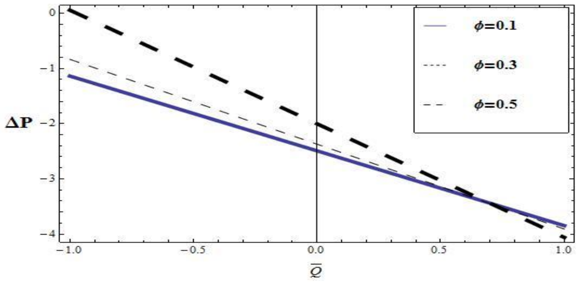


Fig. 4 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'ϕ'.

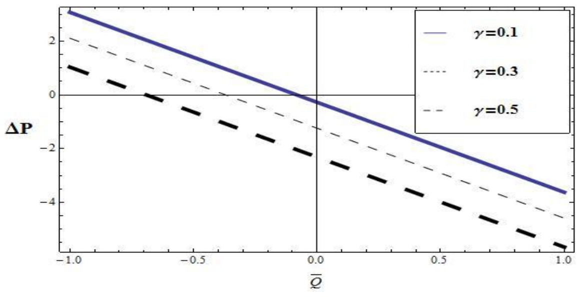


Fig. 5 Pressure rise Δp vs flow rate $\bar{Q}$ for different values of 'γ'.

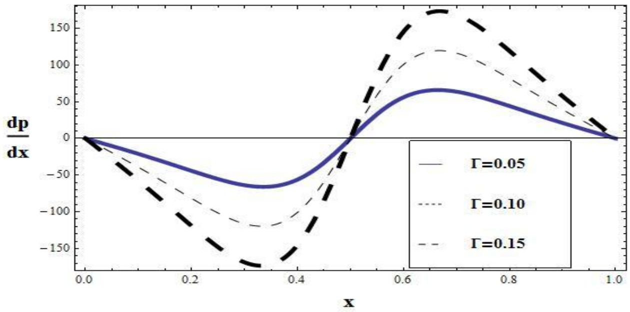


Fig. 6 Pressure gradient $\frac{dp}{dx}$ vs x for different values of 'Γ'.

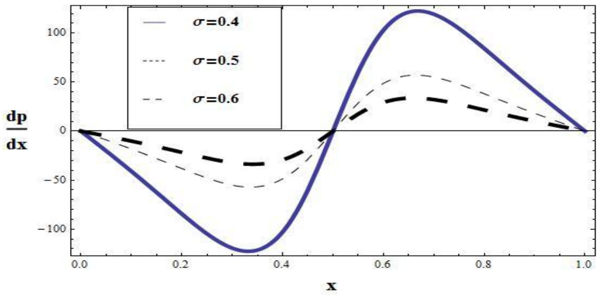


Fig. 7 Pressure gradient $\frac{dp}{dx}$ vs x for different values of 'σ'.

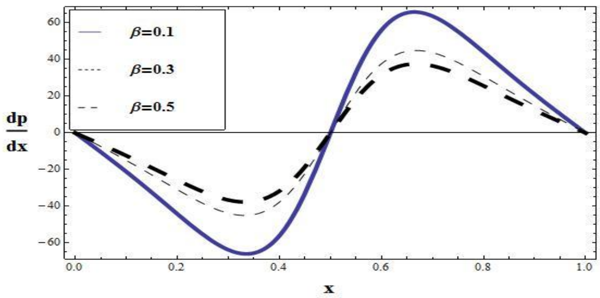


Fig. 8 Pressure gradient $\frac{dp}{dx}$ vs for different values of 'β'.

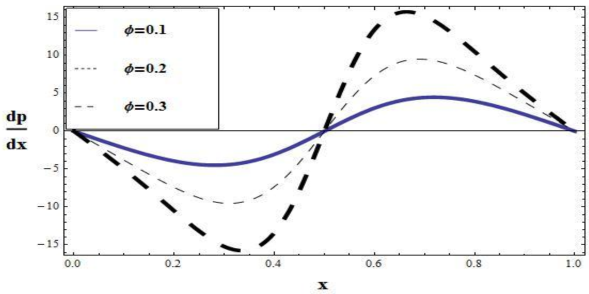


Fig. 9 Pressure gradient $\frac{dp}{dx}$ vs x for different values of 'ϕ'.

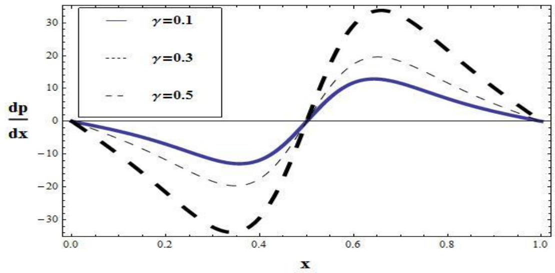


Fig. 10 Pressure gradient $\frac{dp}{dx}$ vs x for different values of 'γ'.

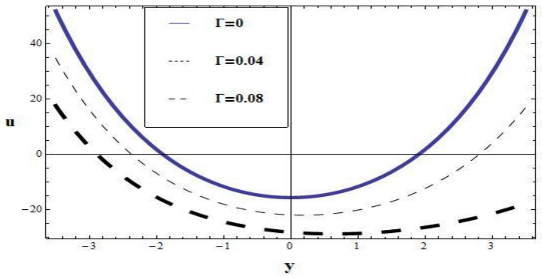


Fig. 11 Axial velocity u vs y for different values of 'Γ'.

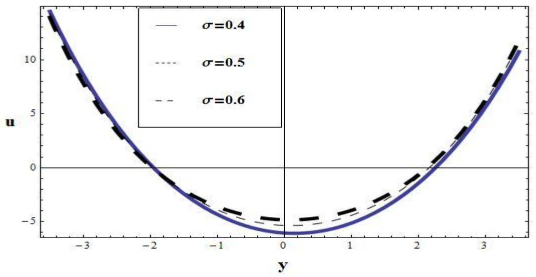


Fig. 12 Axial velocity u vs y for different values of 'σ'.

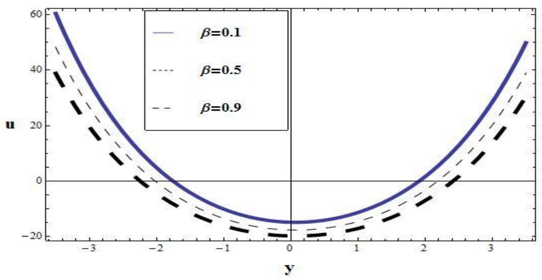


Fig. 13 Axial velocity u vs y for different values of 'β'.

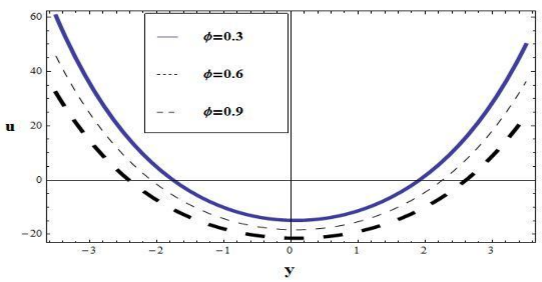


Fig. 14 Axial velocity u vs y for different values of 'ϕ'.

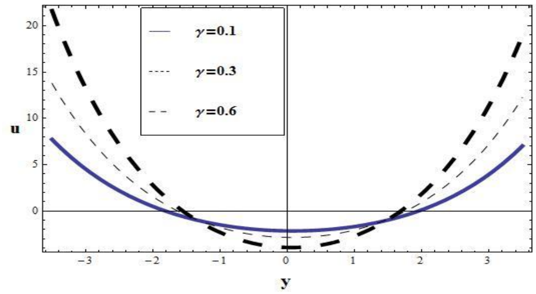
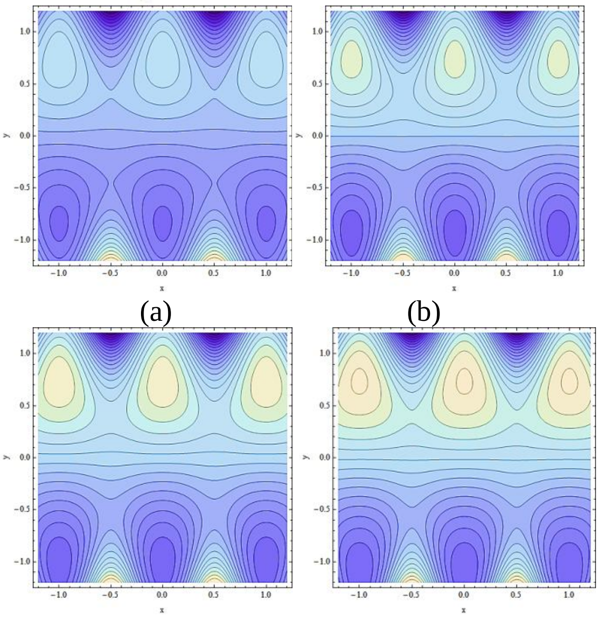


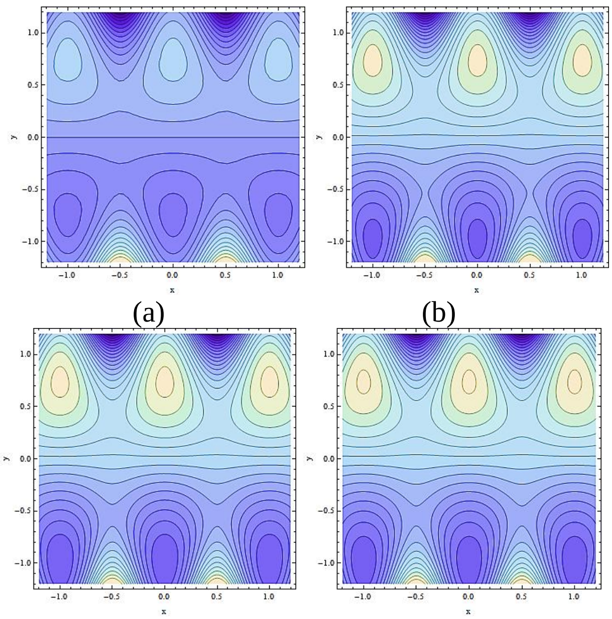
Fig. 15 Axial velocity u vs y for different values of 'γ'with



(a) (b)

(c) (d)

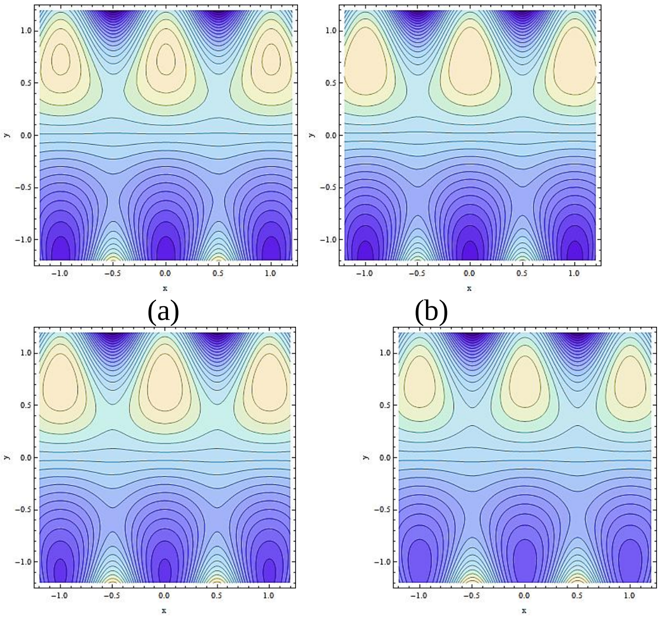
Fig. 16 Stream lines for different values of (a) $\beta=0$ (b) $\beta=0.5$ (c) $\beta=1$ (d) $\beta=1.5$.



(a) (b)

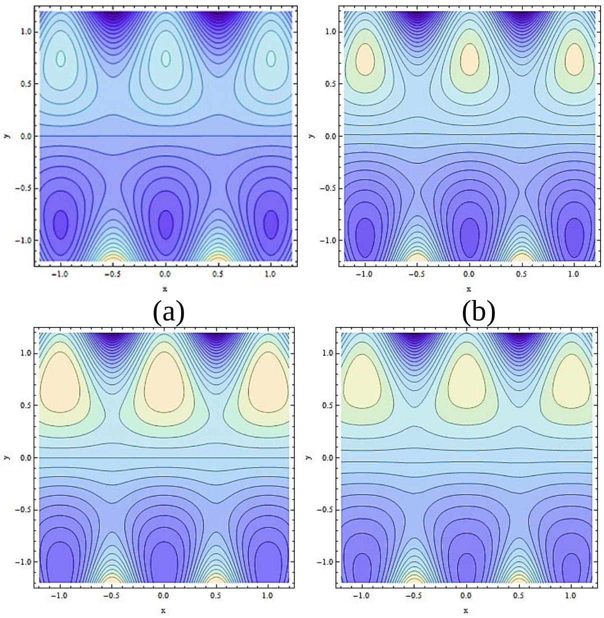
(c) (d)

Fig. 17 Stream lines for different values of (a) $\Gamma=0$ (b) (c) $\Gamma=0.04$ (d)$\Gamma=0.06$.



(a) (b) (c) (d)

Fig. 18 Stream lines for different values of (a) $\gamma = 0.1$ (b) (c) $\gamma = 0.3$ (d) $\gamma = 0.4$.



(a) (b) (c) (d)

Fig. 19 Stream lines for different values of (a) $\sigma = 0.4$ (b) $\sigma = 0.6$ (c) $\sigma = 0.8$ (d) $\sigma = 1$.

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