



Natural convection flow in a vertical channel with inclined magnetic field and Soret effects

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Abstract: The thermal diffusion and inclined magnetic field effects on free convection flow through a channel are investigated. This study includes the influence of hall current also. Homotopy Analysis Method (HAM) is applied to get solution of the dimensionless governing equations those were transformed by similarity transformations from the system of governing partial differential equations. Influence of all the emerging numbers of this study on all the profiles were presented through plots.

Keywords: Natural convection, Inclined magnetic field, Soret effect, HAM.

I. INTRODUCTION

Natural convection flow between vertical parallel plates with heat mass transfer has a remarkable significance in various fields. The significance and developments of heat and mass transfer have been addressed many researchers (Srinivasacharya and Kaladhar, 2012; Ashmawy, 2015; Terekhov et.al, 2016). In view of applications, Umavathi et al. (2016) studied the variable viscosity and thermal conductivity effects on free convection flow of a viscous fluid between vertical parallel plates. Most recently, Kaladhar and Komuraiah (2017) considered the free convection flow between vertical parallel plates with Navier slip, Soret and Dufour effects.

Many researchers considered the magnetic field vertical to the plates. But, in many realistic circumstances such magnetic field may not suitable always. Also, the influence of Hall current becomes most significant mechanism for the electrical conduction in ionized gases and plasmas when the applied magnetic field is strong. The importance and past contributions can be seen in (Ghosh et.al, 2010; Sarkar et.al, 2014; Kothandapani and Prakash, 2015; Abbasi et.al, 2015). Recently, Ajaz and Elangovan (2017) reported the impact of an inclined magnetic field in an inclined channel saturated with peristaltic flow of a couple stress fluids. Most recently, Mishra and Sharma (2017) examined the effects of inclined magnetic field with the

hall current on mixed convection flow in a revolving channel.

The Soret effect encountered in many areas for instance geosciences and chemical engineering, etc.,. For the significance and past literature one can refer in (Mahmoud and Megahed, 2013; Roy and Murthy, 2015; Khan et.al, 2016). In view of the importance most recently, Motsa et al. (2017) used new bivariate pseudo-spectral local linearization method to find the Soret effect on natural convection over the vertical frustum of a cone in a nanofluid.

In this paper, the impact of hall current, inclined magnetic field and diffusion thermo on natural convection flow through a vertical channel has been examined. The survey clearly shows that the present study has not been reported elsewhere. Due to the significance, the authors are enthused to come with this study. Homotopy analysis method (HAM) (Liao, 2003; Kaladhar and Komuraiah, 2018) has been applied to solve this problem. Then the nature of all the profiles against to the pertinent dimensionless numbers of this paper has been presented through graphs.

II. MATHEMATICAL MODELLING

Three-dimensional free convection flow in a channel has been considered. Flow constitution is explained in Fig. 1. A constant external magnetic field B_0 is applied

in the direction which makes an angle α with the positive direction of x -axis. All the properties of the fluid are chosen to be fixed with the exclusion that the density in the buoyancy term. All the flow variables are functions of y only; this is because of the plates extended infinitely in x direction. The governing three-dimensional steady flow equations are in the form:

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} + \rho g^* [\beta_T (T - T_1) + \beta_C (C - C_1)] - \frac{\sigma B_0^2 \cos \alpha}{1+m^2 \cos^2 \alpha} [u \cos \alpha - v_0 \sin \alpha + m w \cos^2 \alpha] \quad (1)$$

$$\rho v_0 \frac{\partial w}{\partial y} = \mu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2 \cos^2 \alpha}{1+m^2 \cos^2 \alpha} [m u \cos \alpha - w - m v_0 \sin \alpha] \quad (2)$$

$$\rho C_p v_0 \frac{\partial T}{\partial y} = K_f \frac{\partial^2 T}{\partial y^2} + 2\mu \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \quad (3)$$

$$v_0 \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

with

$$u(-d) = w(-d) = 0, T(-d) = T_1, C(-d) = C_1, u(d) = w(d) = 0, T(d) = T_2, C(d) = C_2 \quad (5)$$

where u, v, w are the velocities in x, y and z directions respectively. $T_m, C_p, K_f, \rho, D, \mu, \beta_T, g^*, \beta_C$ are the mean fluid temperature, specific heat, thermal diffusion ratio, density, mass diffusivity, coefficient of viscosity, coefficient of thermal expansion, acceleration due to gravity and coefficient of solutal expansion respectively.

Introducing the following dimensionless variables

$$y = \eta d, f = \frac{d}{\gamma Gr} u, g = \frac{d}{\gamma Gr} w, T - T_1 = (T_2 - T_1) \theta, C - C_1 = (C_2 - C_1) \phi \quad (6)$$

in Eqs. (1) - (5), dimensionless equations of the governing system obtained as

$$f'' - Re f' + \theta + N \phi - \frac{Ha^2 \cos \alpha}{1+m^2 \cos^2 \alpha} [f \cos \alpha - \frac{Re}{Gr} \sin \alpha + m g \cos^2 \alpha] = 0 \quad (7)$$

$$g'' - Re g' + \frac{Ha^2 \cos \alpha}{1+m^2 \cos^2 \alpha} [m f \cos \alpha - g - \frac{m Re}{Gr} \sin \alpha] = 0 \quad (8)$$

$$\theta'' - Re Pr \theta' + 2 Br Gr^2 [(f')^2 + (g')^2] = 0 \quad (9)$$

$$\phi'' - Re Sc \phi' + Sr Sc \theta'' = 0 \quad (10)$$

with

$$f(-1) = g(-1) = \theta(-1) = \phi(-1) = 0, f(1) = g(1) = 0, \theta(1) = \phi(1) = 1 \quad (11)$$

where the primes represents differentiation with respect

to η , $Re = \frac{v_0 d}{\nu}$ is the Reynolds number, $Gr = \frac{g^* \beta_T (T_2 - T_1) d^3}{\nu^2}$ is the temperature Grashof number, $Pr = \frac{\mu C_p}{K_f}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the

Schmidt number, $Br = \frac{\mu \nu^2}{K_f d^2 (T_2 - T_1)}$ is the

Brinkman number, $Sr = \frac{DK_T (T_2 - T_1)}{\nu T_m (C_2 - C_1)}$ is the Soret number, $N = \frac{\beta_C (C_2 - C_1)}{\beta_T (T_2 - T_1)}$ is the buoyancy parameter,

$Ha = dB_0 \sqrt{\frac{\sigma}{\nu}}$ is the Hartmann number and m is the hall number.

Effects of the different emerging parameters present in this study on physical quantities are presented in the following section.

III. SOLUTION OF THE PROBLEM

To obtain HAM solutions, the initial guess and the auxiliary linear operators for $f(\eta), g(\eta), \theta(\eta)$ and $\phi(\eta)$ are considered as

$$f_0(\eta) = 0, g_0 = 0, \theta_0(\eta) = \frac{1+\eta}{2}, \phi_0(\eta) = \frac{1+\eta}{2}, \quad L = \frac{\partial^2}{\partial \eta^2} \text{ such that } L(c_1 + c_2 \eta) = 0 \quad (12)$$

Zeroth-order deformations are initiated and in which h_1, h_2, h_3 and h_4 (convergence control parameters) were introduced.

Homotopy solutions were obtained by considering N_1, N_2, N_3 and N_4 (non-linear operators); total average residual errors for f, g, θ and ϕ are considered as presented in (Kaladhar and Komuraiah, 2018). h -curves are plotted to decide the suitable (Optimal) values of h_1, h_2, h_3 and h_4 by taking $Re=2.0, m=2, Pr=0.71, Ha=2, Sc=0.22, N=2.0, Br=0.1, Gr=0.5, Sr=2.0$. From the h -curves and the average residual errors, the suitable values for h_1, h_2, h_3 and h_4 are fixed as -0.79, -1.08, -0.81, -1.38 respectively.

IV. DISCUSSION OF RESULTS

In the absence of Hartmann number Ha , buoyancy parameter N and Grashof number Gr , in equation (7) and (9), the analytical solution of those equations with

boundary conditions (11) and the Homotopy solution are presented through figures. The comparisons are found to be in a very good agreement. Therefore, the HAM code can be used with great confidence to study the problem considered in this paper.

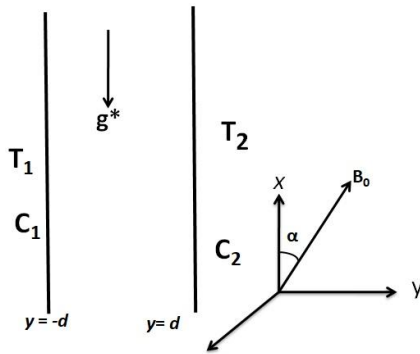


Figure 1: Geometry of the problem.

The influence of Ha , m , α , Sr on velocities ($f(\eta)$, $g(\eta)$), temperature ($\theta(\eta)$) and concentration ($\phi(\eta)$) are calculated and are explained Figs. 6 to 21 by fixing Pr , Br , Re , Sc , Gr , N at 0.71, 0.5, 2, 0.22, 0.5, 2 respectively.

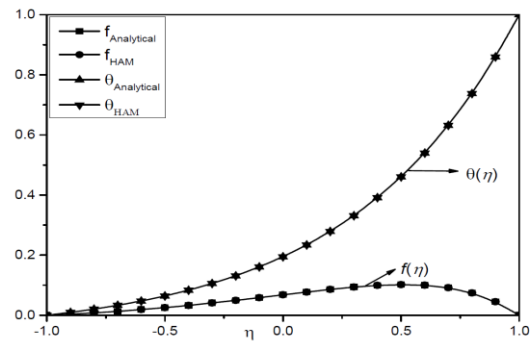


Figure 2: Comparison of flow velocity (f) and temperature (θ) with analytical solutions

Figures 7-10 shows the impact of Hartman number (Ha) on f , g , θ and ϕ when $\alpha=\pi/3$, $Sr=2$, $m=2$. It is seen from Fig.7 that as Ha increases, the flow velocity increases. It is noted that the applied magnetic field has an inclination angle $\alpha>0$ with that the drag force cannot

be generated. It is identified from Fig. 8 that the cross flow velocity increases as an increase in Ha . It can depict from Figs. 9-10 that the dimensionless temperature enhances and concentration diminishes with the increase of magnetic parameter.

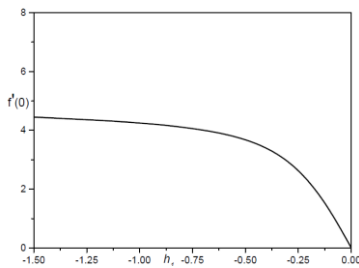


Figure 3: h curves for $f(\eta)$

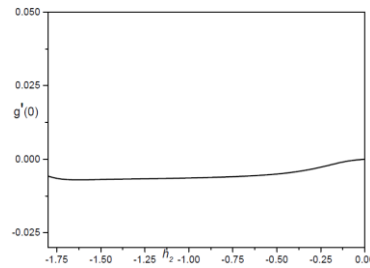


Figure 4: h curves for $g(\eta)$

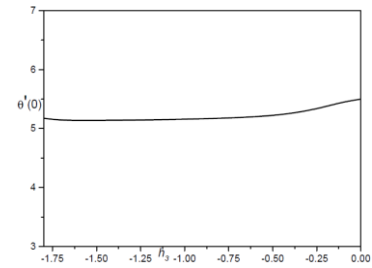


Figure 5: h curves for $\theta(\eta)$

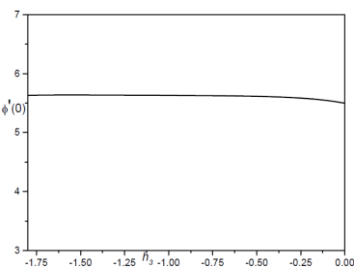


Figure 6: h curves for $\phi(\eta)$

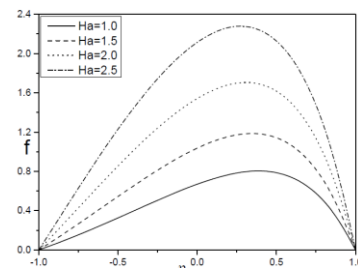


Figure 7: Magnetic parameter effect on $f(\eta)$

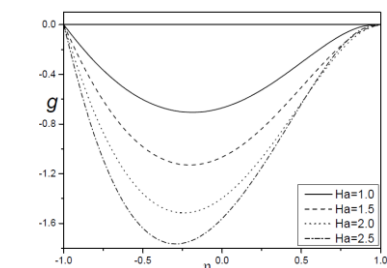


Figure 8: Ha effect on $g(\eta)$

The influence of m on f , g , θ and ϕ can be found in Figs. 11 to 14 at $Ha=2$, $\alpha=\pi/3$, $Sr=2$. It is noticed from Figs. 11-12 that the flow velocity decreases and the cross flow velocity increases as m magnifies. It is noted from Figs. 13-14 that the temperature of the fluid decreases and the concentration of the fluid increases as an increase in m . When there is a magnetic field acting with an angle $\alpha=\pi/3$, hall current will be generated perpendicular to both the direction and which will act as drag on flow velocity and temperature. As explained above the hall current generates extra charge and which leads to

increase in cross flow velocity and the concentration of the fluid.

The effect of an inclination angle α on f , g , θ and ϕ can be noted in Figs. 15-18 by fixing the other parameters at $Ha=2$, $m=2$, $Sr=2$. It is noticed from Figs. 15 and 16 that the flow velocity and cross flow velocity increases as α increase. This is due to the reason that as an inclination angle increases the direction of the applied magnetic field changes and the drag force will reduce on the net flow. It is observed from Fig. 17-18 that the

dimensionless temperature increases and concentration decreases as α increase.

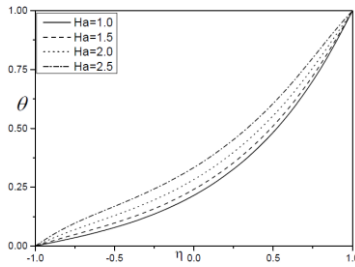


Figure 9: Magnetic parameter effect on $\theta(\eta)$

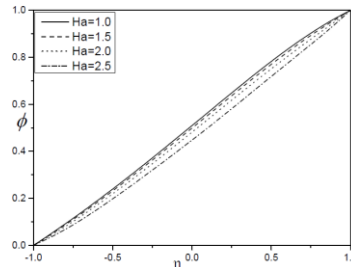


Figure10: Ha effect on $\phi(\eta)$

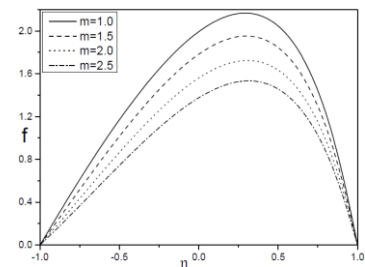


Figure11: Hall effect on $f(\eta)$

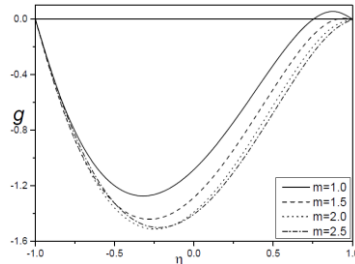


Figure12: Hall effect on $g(\eta)$

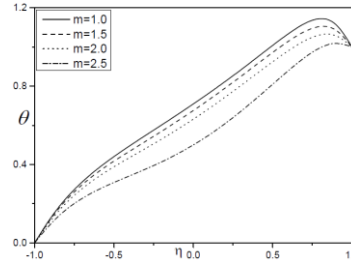


Figure13: Hall effect on $\theta(\eta)$

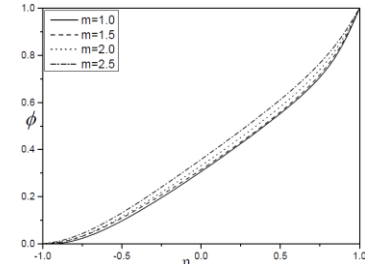


Figure14: Hall effect on $\phi(\eta)$

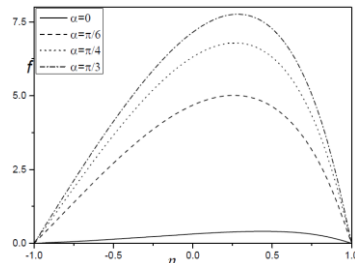


Figure15: Influence of α on $f(\eta)$

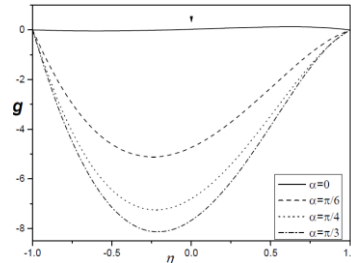


Figure16: Influence of α on $g(\eta)$

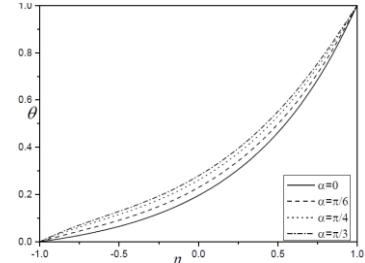


Figure17: Influence of α on $\theta(\eta)$

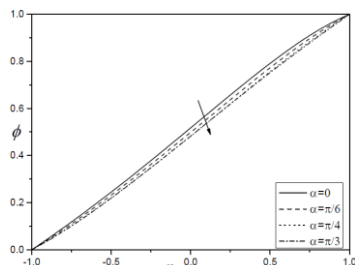


Figure18: Influence of α on $\phi(\eta)$

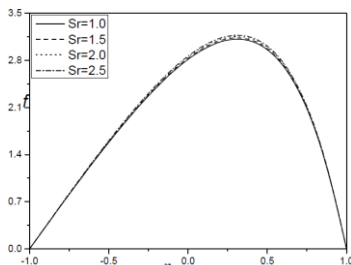


Figure19: Influence of Sr on $f(\eta)$

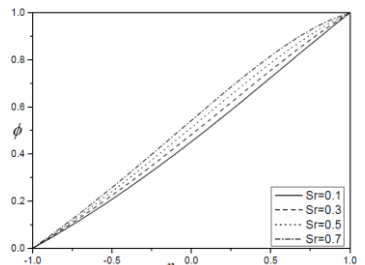


Figure20: Influence of Sr on $\phi(\eta)$

Figures19-20 presents the influence of Soret parameter on the flow velocity $f(\eta)$ and concentration $\phi(\eta)$ at $Ha=2$, $m=2$, $\alpha=\pi/3$. It is noted from Figs.19-20 that the flow velocity and the concentration enhance as the Soret parameter increases. These results clearly disclose that the flow field is appreciably influenced by the Soret parameter.

V. CONCLUSION

The present investigation presents the steady inclined magnetohydrodynamic fluid flow in a vertical channel with Hall and diffusion thermo effect. Homotopy method is applied to solve the final system. It is noted from this present analysis that the velocities and the temperature of the fluid amplifies but the concentration of the fluid flow reduces with the increase of Ha and α . As m enhances, f and θ decreases but reverse trend is

observed in case g and ϕ . It is found that the flow velocity and the concentration magnifies as Sr increases.

Conflict of Interest

Both the authors have equal contribution in this work and it is declared that there is no conflict of interest for this publication.

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References

- [1] Abbasi, F. M., Hayat, T., &Alsaedi, A. (2015). Effects of inclined magnetic field and Joule heating in mixed convective peristaltic transport of non-Newtonian fluids. *Bulletin of the polish academy of sciences technical sciences*, 63(2), 501-514.
- [2] Ashmawy, E. A. (2015). Fully developed natural convective micropolar fluid flow in a vertical channel with slip. *Journal of the Egyptian Mathematical Society*, 23(3), 563-567.
- [3] Dar, A. A., &Elangovan, K. (2017). Influence of an inclined magnetic field on heat and mass transfer of the peristaltic flow of a couple stress fluid in an inclined channel. *World Journal of Engineering*, 14(1), 7-18.
- [4] Ghosh, S. K., Bég, O. A., &Zueco, J. (2010). Hydromagnetic free convection flow with induced magnetic field effects. *Meccanica*, 45(2), 175-185.
- [5] Kaladhar, K., & Komuraiah, E. (2017). Homotopy analysis for the influence of Navier slip flow in a vertical channel with cross diffusion effects. *Mathematical Sciences*, 11(3), 219-229.
- [6] Kaladhar, K., & Komuraiah, E. (2018). Influence of cross diffusions on mixed convection chemical reaction flow in a vertical channel with navier slip: Homotopy approach. *Journal of Applied Analysis and Computation*, 8(1), 379-389.
- [7] Khan, U., Ahmed, N., & Mohyud-Din, S. T. (2016). Soret and Dufour effects on flow in converging and diverging channels with chemical reaction. *Aerospace Science and Technology*, 49, 135-143.
- [8] Liao, S. (2003). *Beyond perturbation: introduction to the Homotopy analysis method*. Chapman and Hall/CRC.
- [9] Mahmoud, M. A. A., &Megahed, A. M. (2013). Thermal radiation effect on mixed convection heat and mass transfer of a non-Newtonian fluid over a vertical surface embedded in a porous medium in the presence of thermal diffusion and diffusion-thermo effects. *Journal of Applied Mechanics and Technical Physics*, 54(1), 90-99.
- [10] Mishra, A., & Sharma, B. K. (2017). MHD Mixed Convection Flow in a Rotating Channel in the Presence of an Inclined Magnetic Field with the Hall Effect. *Journal of Engineering Physics and Thermophysics*, 90(6), 1488-1499.
- [11] Motsa, S. S., RamReddy, C., & Rao, C. V. (2017). Non-similarity solution for Soret effect on natural convection over the vertical frustum of a cone in a nanofluid using new bivariate pseudo-spectral local linearisation method. *Applied Mathematics and Computation*, 314, 439-455.
- [12] Roy, K., & Murthy, P. V. S. N. (2015). Soret effect on the double diffusive convection instability due to viscous dissipation in a horizontal porous channel. *International Journal of Heat and Mass Transfer*, 91, 700-710.
- [13] Sarkar, B. C., Das, S., & Jana, R. N. (2014). Hall Effects on MHD Flow in a Rotating Channel in the Presence of an Inclined Magnetic Field. *Journal of Applied Science and Engineering*, 17(3), 243-252.
- [14] Srinivasacharya, D., &Kaladhar, K. (2012). Mixed convection flow of couple stress fluid between parallel vertical plates with Hall and Ion-slip effects. *Communications in Nonlinear Science and Numerical Simulation*, 17(6), 2447-2462.
- [15] Terekhov, V. I., Ekaid, A. L., & Yassin, K. F. (2016). Laminar free convection heat transfer between vertical isothermal plates. *Journal of Engineering Thermophysics*, 25(4), 509-519.
- [16] Umavathi, J. C., Chamkha, A. J., & Mohiuddin, S. (2016). Combined effect of variable viscosity and thermal conductivity on free convection flow of a viscous fluid in a vertical channel. *International Journal of Numerical Methods for Heat & Fluid Flow*, 26(1), 18-39.